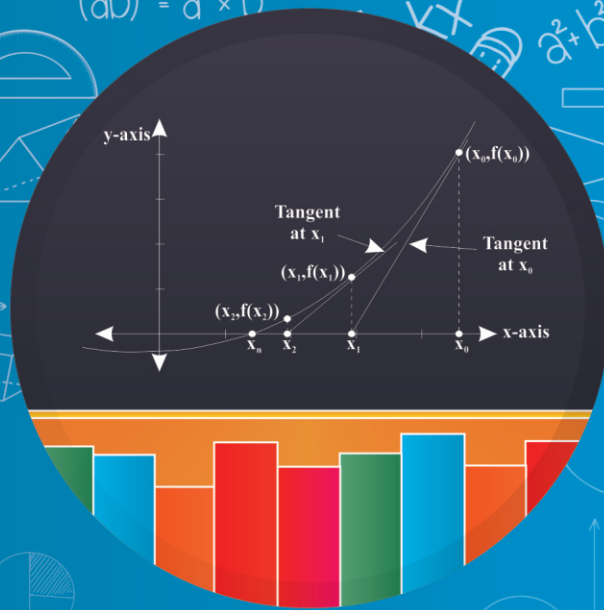


As per National Education Policy (NEP)-2020



A Textbook on Numerical Methods and Analysis



Prof. Chaitanya Kumar
Dr. Harinderjit Kaur Chawla
Dr. Indarpal Singh

Sultan Chand & Sons

Numerical Methods and Analysis

***Dedicated to the memory of
our Loving Grand Parents***

Numerical Methods and Analysis

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Preface

The authors take immense pleasure in presenting to the readers the book entitled “Numerical Methods and Analysis” which includes considerable new material to make the coverage more comprehensive and contemporary.

This book has been designed in accordance with the Undergraduate Curriculum Framework-2022, followed by the Central Universities of India, including University of Delhi under the National Education Policy (NEP)-2020. It is exclusively crafted to cater to the interest of students of B.Sc. (Hons.) Mathematics DSC-12 Semester-IV; GE5(i) B.Sc./B.A. (Hons.) (other than Mathematics) Semester-V; B.Sc. (Physical Sciences/ Mathematical Sciences) DSE-1(b) Semester-V; GE-5(i) Semester-V and DSC-13 Semester-VII Bachelor in Multidisciplinary courses with 2 core and 3 core disciplines; B.Sc.(H) Operational Research DSE-1(b) Semester-V and B.Sc. (H) Statistics DSE-09 Semester-V. It is also useful for B.Tech students of various Universities and for the preparation of competitive examinations. The students of open and distance education courses will also find the book extremely useful.

A sincere and humble attempt has been made to provide the readers a complete and self-sufficient book written in a lucid and simplified manner, giving comprehensive step-by-step explanations for better understanding of the subject.

There are 8 chapters in this book. In each chapter the concepts are vividly explained, supported by nearly 153 Examples and 105 Exercise Questions to provide the students an integrated view of the theory and its applications. A separate chapter comprising Practicals of Numerical Analysis using the software *Mathematica* is also provided to emphasize the importance of qualitative analysis of the systems. Finally, recent University of Delhi Question Papers with their solutions have been included for ample practice.

Most of the questions conform to the examination pattern followed in the University Examinations and Professional Examinations.

We gratefully acknowledge the inspiration, encouragement and valuable suggestions received from the teachers who are teaching undergraduate and postgraduate courses in several Universities. In particular we convey our thanks to Prof. Rajiv Chopra (Principal,

DCAC, University of Delhi) Prof. Suman Sharma (Principal, Lady Shri Ram College for Women, University of Delhi), Prof. Haritma Chopra (Officiating Principal, Maitreyi College, University of Delhi), Prof. Ajay Kumar (NASI, Senior Scientist), Prof. R.K. Mohanty (Officiating President, South Asian University), Prof. Ayub Khan (ex-Head of Department of Mathematics, Jamia Millia Islamia), Prof. Ruchi Das (Head of Department of Mathematics, University of Delhi), Prof. J.K. Singh (Head of Department of Mathematics, Netaji Subhash University of Technology New Delhi), Prof. C.K. Jaggi (Department of Operations Research, University of Delhi), Prof. Arshad Khan, Jamia Millia Islamia, Prof. Om Pal Singh (Department of Operations Research, University of Delhi), Prof. Rajiv Agarwal (Principal, Deshbandhu College, University of Delhi), Prof. S.K. Verma, (Head of Department at SOL, University of Delhi), Prof. Kul Anand Sharma, Professor B.K. Tyagi, Dr. Satyendra Kumar, Dr. Arun Bala Vaish, Dr. Chaman Singh, Dr. Bhavneet Kaur, Dr. Geetan Manchanda, Sumit Mittal (DCAC, University of Delhi).

Gratitude is also due to our families for their unwavering support throughout this journey. Our heartfelt acknowledgement is due to the entire staff of the publisher, Sultan Chand & Sons and their signs for the efficiency, assistance and wholehearted cooperation. Genuine efforts have been made to proofread the book to avoid any errors. However, any mistakes left out inadvertently brought to our notice will be thankfully acknowledged.

New Delhi

Prof. (Dr.) Chaitanya Kumar
Dr. Harinderjit Kaur Chawla
Dr. Indarpal Singh

Syllabus

B.Sc. (Hons.) Mathematics Semester-IV, DSC-12

UNIT – I: Methods for Solving Algebraic and Transcendental Equations

Rate and order of convergence; Bisection method, Method of false position, Fixed point iteration method, Newton's method, and Secant method, their order of convergence and convergence analysis.

UNIT – II: Techniques to Solve Linear Systems and Interpolation

LU decomposition and its applications; Iterative methods: Gauss–Jacobi, Gauss–Seidel methods; Lagrange and Newton interpolation, Piecewise linear interpolation.

UNIT – III: Numerical Differentiation and Integration

First and higher order approximation for the first derivative, Approximation for the second derivative; Numerical integration by closed Newton–Cotes formulae: Trapezoidal rule, Simpson's rule and its error analysis; Euler's method to solve ODE's, Modified Euler method, Runge–Kutta Method (fourth-order).

B.Sc./B.A. (Hons) (other than Mathematics) GE-5(i)

UNIT-I: Errors and Roots of Transcendental and Polynomial Equations

Errors: Roundoff error, Local truncation error, Global truncation error; Order of a method, Convergence, and terminal conditions; Bisection method, Secant method, Regula–Falsi method, Newton–Raphson method.

UNIT-II: Algebraic Linear Systems and Interpolation

Gaussian elimination method (with row pivoting); Iterative methods: Jacobi method, Gauss–Seidel method; Interpolation: Lagrange form, Newton form, Finite difference operators.

UNIT-III: Numerical Differentiation, Integration and ODE

First and second order numerical derivatives; Trapezoidal rule, Simpson's rule for numerical integration; Ordinary differential equation: Euler's, and Runge-Kutta method.

**B.Sc. (Hons) Operation Research
Semester-V DSE-1(b)****Unit I**

Errors: Relative Error, Absolute Error, Round off Error, Truncation Error. Transcendental and Polynomial equations: Bisection method, Newton-Raphson method, Secant method, Method of False Position, Fixed point iterative method, Order and rate of convergence of these methods.

Unit II

System of linear equations: Gauss Elimination and Gauss Jordan methods, Gauss Jacobi method, Gauss Seidel method and their convergence analysis.

Unit III

Interpolation: Lagrange Interpolating Polynomial, Newton's Gregory forward and backward difference interpolating polynomial, Newton's Divided Difference Interpolating Polynomial, Error analysis in each method.

Unit IV

Numerical Integration: Trapezoidal rule, Composite Trapezoidal rule, Simpson's rule, Composite Simpson's rule, Simpsons 3/8th rule Ordinary Differential Equations: Euler's method, Modified Euler's method, Runge-Kutta method



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Snapshot of the Book

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2.	Methods for Solving Algebraic and Transcendental Equations	42	32	23	8
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4.	Finite Differences and Interpolation	20	15	17	–
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7.	Numerical Solution of Ordinary Differential Equations	18	12	12	1
8.	Practicals in Numerical Analysis	50	48	22	17
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	University Question Paper-2022	11	–	–	–
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	Total	232	153	105	26

About the Book

This book has been designed in accordance with the Undergraduate Curriculum Framework-2022, followed by the Central Universities of India, including University of Delhi under the National Education Policy (NEP)-2020. It is exclusively crafted to cater to the interest of students of B.Sc. (Hons.) Mathematics DSC-12 Semester-IV; GE-5(I) B.Sc./B.A. (Hons.) (other than Mathematics) Semester-V; B.Sc. (Physical Sciences/Mathematical Sciences) DSE-1(b) Semester-V; GE-5(I) Semester-V and DSC-13 Semester-VII Bachelor in Multidisciplinary courses with 2 core and 3 core disciplines; B.Sc.(Hons.) Operational Research DSE-1(b) Semester-V and B.Sc. (Hons.) Statistics DSE-09 Semester-V. It is also useful for B.Tech students of various Universities and for the preparation of competitive examinations. The students of open and distance education courses will also find the book very beneficial.

Salient Features

- An all-encompassing and self-sufficient textbook for UGCF-2022 based on NEP-2020.
- Written in lucid and simple language.
- Written with a view to present a qualitative understanding of the subject.
- Comprehensive step-by-step explanations for easier understanding of the subject.
- Many solved examples and unsolved problems have been drawn from recent examination papers of Universities.
- Answers to all problems in each exercise have been provided immediately after the exercise for the convenience of the reader.
- Recent Delhi University Question Papers with Solutions have been included for ample practice.

About the Authors

Prof. Chaitanya Kumar, M.Sc., M.Phil., Ph.D., University of Delhi, has been a Professor of Mathematics, Delhi College of Arts and Commerce, University of Delhi. He has published several research papers in the area of boundary value problems and integral equations in reputed International Journals. He has been a Research Supervisor at Post Graduate and higher levels.



He has authored 10 important books on Mathematics: *Essentials of Mathematics for Business Studies*, *Mathematics for M.C.A. Entrance Examination*, *Basic Research Methods and Statistics for Social Science*, *Theory of Analytic Geometry and Applied Algebra*, *Elements of Analysis*, *Ordinary and Partial Differential Equations*, *Probability and Statistics*, *Fundamentals of Calculus*, *Analytical Geometry and Differential Equations and Applications* for graduate and higher levels.



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