

A circular gold emblem with a blue center. The center contains the text "10th" in white, flanked by three white stars above and three white stars below. A gold ribbon banner wraps around the bottom of the circle, containing the word "Edition" in a blue serif font.

B.C. Mehta • G.M.K. Madnani



Sultan Chand & Sons

MATHEMATICS FOR ECONOMISTS

B.C. Mehta **G.M.K. Madnani**
Professor *Professor*



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Author's Acknowledgement: The writing of a Textbook always involves creation of a huge debt towards innumerable authors and publications. We owe our gratitude to all of them. We acknowledge our indebtedness in extensive footnotes throughout the book. If, for any reason, any acknowledgement has been left out we beg to be excused. We assure to carry out correction in the subsequent edition, as and when it is known.

Preface

To the Tenth Edition

It is with immense pleasure and gratitude that we present the Tenth Edition of Mathematics for Economists. Since the publication of the very first edition in July 1973, this book has embarked on a remarkable journey, continually evolving to meet the dynamic and ever-expanding needs of students, researchers, and educators across India. For over fifty years, our primary objective has remained completely steadfast: to demystify the quantitative tools that form the absolute bedrock of modern economic theory. We have always strived to provide a clear and structured pathway for students to master the mathematical techniques that are undeniably essential for their academic progression and future professional success.

As noted in our inaugural edition, the mathematisation of economics was a revolutionary shift; today, it is an absolute standard. The modern economic landscape is increasingly data-driven, defined by complex programming, rigorous estimation, and dynamic modelling using AI. Whether dealing with consumer equilibrium, input-output analysis, or the time-paths of national income in growth economics, mathematics serves as the essential short-hand of logic. It transforms abstract economic laws into precise, actionable, and empirical relationships. This core philosophy continues to guide the pedagogical structure of this volume.

The transition to this tenth edition was driven by comprehensive feedback from our dedicated community of teachers and the evolving assessment patterns of modern universities. In an era where competitive examinations and academic grading demand not just conceptual clarity but also swift problem-solving abilities, a textbook must serve as both a theoretical guide and a rigorous practice ground. With a sharp focus on student assessment and practical application, this revised edition introduces several significant pedagogical and structural improvements:

1. **Comprehensive Question Bank (230 MCQs):** To align with contemporary examination formats and foster quick recall of fundamental concepts, we have integrated a robust repository of 230 Multiple Choice Questions to aid in competitive exam preparation.
2. **Model Question Papers:** These serve as self-assessment tools, allowing students to simulate exam conditions, manage their time effectively, and evaluate their comprehensive grasp of the syllabus.
3. **Latest University Question Papers:** To effectively bridge the gap between theoretical classroom learning and actual university evaluations, we have incorporated 17 of the most recent university question papers. This extensive practice material is drawn from various prominent universities, ensuring that students are well-acquainted with the diverse formats, the frequency of specific topics, and the varying levels of complexity typically expected by examiners.
4. **Enhanced Typography and Layout:** Recognizing that complex mathematical formulas and economic models require visual clarity, this edition is now printed in a larger, more accommodating physical size. The typography has been entirely reset to ensure a superior, logical flow of matter.
5. **Extensive Solved Examples and Exercises:** The book is enriched with more than 550 solved examples, over 1100 exercise questions, and around 180 figures for better understanding of the text.

While the practice materials and physical presentation have been significantly upgraded, the book retains its trusted, self-contained structure.

The longevity and success of this book belong as much to the readers as to the authors. We extend our deepest gratitude to the countless students and esteemed faculty members across universities who have generously shared their

insights, pointed out areas for improvement, and championed this text in their classrooms. Your continued engagement makes each edition sharper and more relevant.

We sincerely hope that the Tenth Edition of Mathematics for Economists will prove to be an even more invaluable resource, empowering the next generation of economists with the analytical rigour required to navigate and shape the future of the discipline. We welcome further suggestions for its continuous improvement.

Editorial Team

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To the First Edition

Economic theory has changed significantly its character in the present century. There have been four major revolutions in Economics: (i) Mathematisation of Economics, (ii) Dynamisation of Economics, (iii) Estimation, and (iv) Programming.

Mathematisation of Economics: Economics is basically quantitative and involves relationships among variables. This necessitates as-well-as facilitates the use of mathematics in Economics. However, disdainful and painful it may be to learn mathematical techniques, one cannot dispense with the use of such techniques in this science. Increased mathematisation of Economics has led to completely new branches of Economics such as Mathematical Economics and Econometrics. Strictly speaking, there is no fundamental difference between Mathematical Economics and Economic Theory. Mathematical Economics puts the literary form of economic theory in mathematical language – the short-hand of logic. In doing so it provides concrete form to economic laws and relationships, and makes it more precise and practical. More than this, use of mathematics helps in systematic understanding of the relationships and in derivation of certain results which would either be impossible through verbal logic or would involve clumsy, complex and circular process.

Illustrations of the use of mathematics in Economics are not far to seek. Economics is the science of choice-making. The consumer makes his choice given his tastes, income, and prices, other things remaining the same. Preference scales and indifference curves can then be constructed. But all these involve use of mathematics, ranging from simple Geometry to Calculus. Consumer's equilibrium is studied in the context of given income and prices, but 'Equilibrium' itself is a mathematical concept derived from Statics and Dynamics.

Maximisation of profits or minimisation of costs by a firm under some constraints is the basic problem discussed in the theory of firm. This problem can also be solved with the help of Differential Calculus, Linear Programming or Theory of Games.

Consumer's equilibrium and equilibrium of the firm involve decisions at the margin and marginal analysis is nothing but extension of differential calculus. We also know that price change on account of output change depends upon elasticity of demand and supply and 'Elasticity' is, in fact, a mathematical concept.

Basic relationships in Micro and Macro economics can usually be put in the form of functions and equations: Quadratic, Simultaneous, Differential and Difference. *For example*, in Growth Economics where the time-paths of national income are to be found out (depending on some crucial factors) difference and differential equations are indispensable. In planning models, sectoral targets are fixed only with the help of Input-Output Analysis and Linear Programming. Determinants and Matrix Algebra are of immense use in such techniques.

Thus, in almost all fields of economics, mathematics is useful. Economic phenomena are so complex and interdependent that some abstractions – *ceteris paribus* assumptions – have to be made. It is often not possible for a student of Economics to understand the effect of many factors simultaneously on a particular economic phenomenon. Mathematics is especially helpful in these situations. A simplified model is first built wherein, relationship between the different variables is established by making assumptions and keeping many factors constant. Thereafter, the *ceteris paribus* assumptions are relaxed one by one and the effects analysed.

The proof of pudding lies, in eating. After going through this book the reader will find out that how easy, smooth and systematic it becomes to understand the economic processes and derivation of various results by learning a little of mathematics. *For example*, the cobweb theorem and the stability conditions can be derived and understood very neatly and briefly in not more than two pages. This cannot be possible without the use of the tools which have been used in this volume.

Dynamisation of Economics: Traditional economics explains the situation at a point of time, whereas actual economy witnesses process of change over time. Hence, 'time' must be accounted for and, therefore, dynamisation is a step towards reality. Growth economics is an outgrowth of dynamisation of Economics.

Estimation: Since economics does not deal with pure theorizing alone but also with the relevance of theory in practice, there must be some methods of testing the correspondence between theory and reality. And here comes a new branch of knowledge, *i.e.*, Econometrics, to our rescue. In Econometrics we use mathematics to specify the relationships

as given by economic theory and apply tools of statistics and theory of probability to test theorems of economic theory, estimate parameters and then predict the results. In the words of J. Tinbergen, *Econometrics is mathematical economics working with measured data to give empirical content to economic theory*.

Programming Research: Problems of maximisation and minimisation are basic to Economics. *For example*, we have to minimise cost of transportation of goods from different godowns to different markets. But against this, maximum capacity of each godown is fixed so also the maximum demand in each of the markets. These are termed as constraints or side conditions. Such problems are solved by programming techniques making economics a complete practical science.

All these revolutions thus involve increasing use of mathematics.

A. Chiang in his book, *“Fundamental Methods of Mathematical Economics”*, rightly considers the mathematical approach as a quick mode of transportation from a set of postulates to a set of conclusions. *For example*, if a person intends to go to a place five miles away, he will definitely prefer driving to walking if he possesses a car unless he has time to kill or wants to exercise his legs. Similarly, a theorist who wishes to get to his conclusions more quickly will find it convenient to drive the vehicle of mathematical techniques appropriate for his particular purpose. But all the same he must know the driving and for this driving lessons are essential. In other words, the theorist must accustom himself with mathematical techniques and their applications which are driving lessons to elucidate the problems of economic theory.

One must also be on guard lest the use of mathematics should turn into a disadvantage. Mathematics, like fire, is a bad master although a good servant. It is also an art and naturally one gets fascinated by its aesthetic values after perfecting its techniques. At times, therefore, addiction to mathematics tempts us to make inappropriate economic assumptions and limit ourselves only to the problems that can be solved mathematically. In this connection we would like to reproduce here the conversation between Prof. Samuelson and his student.

Student: “I am interested in economic theory. I know little mathematics. And when I look at the journals I am greatly troubled. Must I give up hope of being a theorist? Must I learn mathematics?”

Samuelson: “Some of the most distinguished economic theorists, past and present, have been innocent of mathematics. Some of the most distinguished theorists have known some mathematics. Obviously you can become a great theorist without knowing mathematics. Yet it is fair to say that you will have to be that much more clever and brilliant ... Mathematics is neither necessary nor a sufficient condition for a fruitful career in economic theory. It can help.....”*

Thus, use of mathematics can only better good economics; it can never compensate for bad economics.

The book is specifically meant for use by students and teachers in India who have got a rudimentary knowledge of mathematics and as such it is presumed that the readers are acquainted only with elementary Algebra and Geometry.

In the first part we have introduced the highly useful Algebra of Matrices and extended the discussion to cover Input-Output Analysis and Linear Programming. In the second part, Differential Calculus – the Calculus of related changes is given in great detail with separate chapters dealing with applications in the various branches of micro-economic theory. In the third part, Integral Calculus, the technique of deriving the macro from the micro analysis, has been dealt with. Differential and Difference equations are covered in the last four chapters with illustrations from macro-economic theory and the theory of economic growth.

Though we have tried to make the book self-contained, we do not claim that we have achieved perfection. It is almost impossible to compress the entire tool-box provided by mathematics in a single volume intended to be studied in a single academic session. The purpose of this work, then, is just to give an introduction to basic mathematics capable of providing tools for use in economic theory and Econometrics.

This is not a book on economic theory. Hence understanding of economic theory is presumed. Only those who have cared to study their economics well will be benefited by this book; it is in no way a substitute for books on economic theory.

We will be glad to receive suggestions for improvement of the volume from those who may use the book as students or as teachers.

JAIPUR
July 25, 1973

MEHTA
MADNANI

* Paul A. Samuelson, ‘Economic Theory and Mathematics – An Appraisal’, *American Economic Review*, Vol. XLII, No. 2, May 1952, pp. 56-66.

Snapshot

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