

Based on Latest
CBCS Pattern

ORDINARY AND PARTIAL
**DIFFERENTIAL
EQUATIONS**

Dr. Chaitanya Kumar

SULTAN CHAND & SONS

Ordinary and Partial Differential Equations

(Based on Latest Syllabus under CBCS)

Dr. CHAITANYA KUMAR

*Associate Professor, Department of Mathematics
Delhi College of Arts and Commerce, University of Delhi*



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SULTAN CHAND & SONS

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Phones : 011-23243183, 23247051, 23266105, 23277843

Fax : 011-2326-6357

Website : www.sultanchandandsons.com

E-mail : sultanchand74@yahoo.com; info@sultanchandandsons.com

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The writing of a textbook always involves creation of a huge debt towards innumerable authors and publications. We owe our gratitude to all of them. We acknowledge our indebtedness in extensive footnotes throughout the book. If, for any reason, any acknowledgement has been left out we beg to be excused. We assure to carry out the corrections in the subsequent edition, as and when it is known.

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Preface

The primary objective of the textbook is to provide the basic concepts of ordinary and partial differential equations as per the requirement of the students appearing for B.A. (Prog.) Semester-V, B.Sc. (Hons.) (Mathematics and Physics) under CBCS pattern followed by Central Universities of India including University of Delhi. This book covers the entire syllabus of the paper Differential Equations — Generic Elective of IIIrd Semester (GE-3) for all Honours courses other than Mathematics and B.Tech. of various Universities. It is also useful for various competitive examinations and School of Open Learning, University of Delhi.

During my long teaching experience, I have fully understood the need of the students and hence I have taken great care to present the subject matter in the most clear, interesting and complete form from the student's point of view.

There are Eleven Chapters in this book and in each of them, the concepts are properly supported by illustrations followed by several varied types of examples to provide students an integrated view of theory and applications. There are about 247 examples in this book. A large number of self-practice problems and answers have been added in each Chapter to enable students to learn. Most of the questions conform to the examination style followed in the University examinations and professional examinations.

We, gratefully, acknowledge the inspiration, encouragement and valuable suggestions received from the teachers who are teaching undergraduate courses of several Universities.

I must be failing in my duty if I do not express my gratitude to Prof. D.L. Jain and Dr. G.C. Goel, Department of Mathematics, Delhi University who taught me and inculcated interest in me, so that I could teach my students to their satisfaction. I express my sincere thanks to all those individuals who have been a source of inspiration and support both personally and professionally.

At the end, I must thank my publishers who took every step for a flawless publication. But to err is human. So any misprint or inaccuracy, if brought to my notice will be thankfully acknowledged.

New Delhi

Dr. CHAITANYA KUMAR

Syllabus

DSE-2 (ii): Differential Equations

Total Marks: 100 (Theory: 75 and Internal Assessment: 25)

Workload: 5 Lectures, 1 Tutorial (per week)

Credits: 6 (5 + 1)

Duration: 14 Weeks (70 hrs.)

Examination: 3 hours

Course objectives: The course aims at introducing ordinary and partial differential equations to the students and finding their solutions using various techniques with the tools needed to model complex real-world situations.

Course learning outcomes: The course will enable the students to understand:

- (i) Wronskian and its properties.
- (ii) Method of variation of parameters and total differential equations.
- (iii) Lagrange's method, and Charpit's method for solving PDE's of first order.

Course Contents

Unit 1: Ordinary Differential Equations

(Lectures: 40)

First order exact differential equations including rules for finding integrating factors; First order higher degree equations solvable for x , y , p and Clairut's equations; Wronskian and its properties; Linear homogeneous equations with constant coefficients; The method of variation of parameters; Euler's equations; Simultaneous differential equations; Total differential equations.

Unit 2: Linear Partial Differential Equations

(Lectures: 15)

Order and degree of partial differential equations; Concept of linear partial differential equations; Formation of first order partial differential equations; Linear partial differential equations of first order and their solutions.

Unit 3: Non-linear Partial Differential Equations

(Lectures: 15)

Concept of non-linear partial differential equations; Lagrange's method; Charpit's method; Classification of second order partial differential equations into elliptic; Parabolic and hyperbolic through illustrations only.

Teaching Plan

[Paper: DSE-2 (ii): Differential Equations]

Weeks 1 and 2: First order exact differential equations including rules for finding integrating factors.

Weeks 3 and 4: First order higher degree equations solvable for x , y , p and Clairut's equations.

Weeks 5 and 6: Wronskian and its properties, Linear homogeneous equations with constant coefficients.

Week 7: The method of variation of parameters, Euler's equations.

Week 8: Simultaneous differential equations, Total differential equation.

Week 9: Order and degree of partial differential equations. Concept of linear partial differential equations, Formation of first order partial differential equations.

Week 10 and 11: Statement of Theorem 2 with applications, Linear partial differential equations of first order and their solutions.

Week 12: Statements of Theorems 4, 5 and 6 with applications, Concept of non-linear partial differential equations, Lagrange's method.

Weeks 13 and 14: Charpit's method, Classification of second order partial differential equations into elliptic, Parabolic and hyperbolic through illustrations only.

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About the Book

This book has been designed for B.A. (Prog.) III Year, Semester-V, B.Sc. (Hons.) (Mathematics and Physics) and GE-3 (for all honours courses) students of all universities of India following Choice Based Credit System (CBCS).

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- Answers to all the problems in each exercise are given immediately after the exercise.
- This book is based on latest syllabus under CBCS pattern.
- Delhi University latest syllabus, Question papers with their solutions are also provided in the book.

About the Author

Dr. Chaitanya Kumar is an Associate Professor, Department of Mathematics, Delhi College of Arts and Commerce, University of Delhi. He has been teaching mathematics for the last thirty seven years. Dr. Chaitanya Kumar received his Doctorate degree in Mathematics from the University of Delhi. He has published several research papers in the area of Boundary Value problems by Integral equation techniques in reputed national and international journals. He has been Research Supervisor at Post Graduate and higher levels. He has authored important books on mathematics: "*Essentials of Mathematics for Business Studies*", "*Mathematics for M.C.A. Entrance Examination*", "*Basic Research Methods and Statistics for Social Sciences*", "*Theory of Analytic Geometry and Applied Algebra*".



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Phones: 23243183, 23247051, 23277843, 23281876, 23266105

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