

ADVANCED MATHEMATICAL METHODS IN PHYSICS

Savita Gahlaut

SULTAN CHAND & SONS

Advanced Mathematical Methods in Physics

*To
Maa
Naresh
Kshitiz
&
Arahan
the four pillars of my life*

Advanced
Mathematical Methods
in Physics

Savita Gahlaut



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Preface

The mathematical concepts are an absolute necessity for physics students. Various mathematical methods are used to understand, study and develop new theories in all the branches of Physics. In this book some advanced mathematical concepts are explained with examples. All the topics covered in the book are explained considering the requirements of Physics students.

The book is aimed for the Honours and Post graduate syllabi of the Indian universities. It is developed as a textbook for the undergraduate and postgraduate students of physics. The topics covered in the book are equally important for the engineering students too. The research student will also find it useful as a quick reference book.

The book is a result of teaching the topics, covered in the book, for about 10 years. Each topic is explained from the basics to make it a self sufficient book. As the book is primarily meant for physics students, it do not have too many theorems as are found in mathematics books which makes the subject dull. Each concept in the book is explained with the help of examples, solved problems and applications in physics. At the end of each chapter more problems are given for practice.

The topics covered in the book are a part of the syllabi prescribed in different Indian Universities. Matrices, vector Spaces, Linear transformations and Cartesian Tensors are also a part of the syllabus of undergraduate Engineering courses in various Technical Institutes and Universities.

The book is based on the syllabus of the paper 'Advanced Mathematical Physics-I' and also covers Group theory which is a part of the paper 'Advanced Mathematical Physics-II' taught to the 3rd year students of B.Sc(H) Physics, Delhi University. I have been teaching these topics for about 10 years and found that students as well as teachers have not been able to find a book which satisfactorily fulfills the requirements of courses in Physics. Most of the books on these topics are meant for Mathematics students which Physics/Engineering students find difficult to relate to the problems of their field. Some books meant for Physics students discuss these topics briefly and do not give enough weightage required for applications in Physics. Moreover, there is not a single book in the market which covers all these topics in

sufficient details and teachers/students have to refer different books for different topics which are part of one paper.

I hope the readers will find the book helpful in understanding the concepts. Any suggestions to improve the book are welcome from the readers. I shall be thankful to one and all to bring to my notice any inadvertent mistakes in the book.

Dr. Savita Gahlaut

Contents

1. Sets, Relations and Operations	1
1.1 Set	1
1.2 Basic Operations on Sets	3
1.3 Cartesian Product of Sets	6
1.4 Relations	6
1.5 Types of Binary Relations	6
1.6 Functions and Mappings	8
1.7 Binary Operations	9
2. Matrices and Determinants	13
2.1 Matrices	13
2.2 Matrix Algebra	13
2.3 Transpose of a matrix	17
2.4 Trace of a square Matrix	18
2.5 Special Square Matrices	19
2.6 Block Matrices	25
2.7 Elementary Matrices	28
2.8 Echelon Matrices	30
2.9 Determinant	31
2.10 Rank of a Matrix	35
2.11 Matrix Inversion	36
3. Groups	43
3.1 Abstract Systems	43
3.2 Semi-Group	44
3.3 Group	44
3.4 Some Elementary Properties of Groups	48
3.5 Cyclic Group	49
3.6 Group of Permutations	52
3.7 Sub-Group	60
3.7.1 Properties of Sub-groups	61
3.8 Cosets of a Sub-Group	62
3.8.1 Properties of Cosets	62
3.9 Lagrange's Theorem	63
3.10 Normal Sub-Group	64
3.10.1 Normalizer of a Group	65
3.11 Centre of a Group	65
3.11.1 Centralizer of a Group	67
3.12 Relation of Conjugacy in a Group	67
3.13 Conjugate Sub-group	68

4. Homomorphism and Isomorphism	71
4.1 Group Homomorphism	71
4.2 Kernel of Homomorphism	72
4.3 Group Isomorphism	74
4.4 Cayley's Theorem	76
5. Vector Spaces	81
5.1 Fields	81
5.2 Vector Space	82
5.3 Subspace	84
5.4 Linear Combination of Vectors	87
5.5 Linear Dependence and Independence	87
5.6 Basis and Dimensions	90
5.7 Coordinates	94
5.8 Isomorphism of Vector Spaces	95
5.9 Inner Product of Vectors	97
5.9.1 Norm of a Vector	99
5.9.2 Cauchy-Schwarz Inequality	99
5.10 Orthogonality	100
5.10.1 Orthonormal Bases	101
6. Linear Transformations	109
6.1 Definition and Examples	109
6.2 Range Space of a Linear Transformation	112
6.3 Kernel of a Linear Transformation	114
6.4 Operations with Linear Transformations	116
6.5 Vector Space of Linear Transformations	119
6.6 Singular and Non-Singular Transformations	120
7. Linear Transformations and Matrices	127
7.1 Matrix Representation of Linear Transformations	127
7.2 Algebra of Linear Transformations	133
7.3 Change of Basis	135
7.4 Similarity Transformations	138
7.5 Moment of Inertia Matrix	142
7.6 Problems	146
8. Eigenvalue Problems and Diagonalization	149
8.1 Eigenvalues and Eigenvectors	149
8.2 Eigenspace of an eigenvalue	153
8.3 Finding Eigenvalues and Eigenvectors	155
8.4 Some Useful Results	160
8.5 Diagonalization	164
8.6 Simultaneously Diagonalizable Matrices	169
8.7 Coupled Linear Differential Equations	174
8.8 Problems	180
9. Function of Matrices	183
9.1 Cayley-Hamilton Theorem	183
9.2 Minimal Polynomial	187
9.3 Minimal Polynomial and Diagonalizability	190

- 9.4 Norm of a Matrix 193
 - 9.5 Matrix Functions 194
 - 9.6 Evaluating Matrix Functions 200
 - 9.7 Problems 203
- 10. Cartesian Tensors 205
 - 10.1 Coordinate Transformations 205
 - 10.2 Tensors of order one and zero 207
 - 10.3 Tensors of order two 208
 - 10.4 Tensors of higher order 210
 - 10.5 Fundamental Operations with Tensors 211
 - 10.6 Tensor Fields 212
 - 10.7 Kronecker Delta 213
 - 10.8 Alternating Tensor 215
 - 10.9 Cross-Product of Vectors 219
 - 10.10 Triple products 220
 - 10.11 Pseudo Tensors 221
 - 10.12 Geometrical Applications 227
 - 10.13 Rotation Tensor 232
 - 10.14 Isotropic Tensors 234
- 11. Physical Tensors 241
 - 11.1 Tensor of Polarizability 241
 - 11.2 Moment of Inertia Tensor 246
 - 11.3 Tensor of Stress 248
 - 11.4 Strain Tensor 254
 - 11.5 Elastic Constants 260

About the Book

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Salient Features

- Each topic is explained from the basics to make it a self-sufficient book.
- All the concepts are explained with the help of examples and solved problems.
- Unsolved problems are given at the end of each chapter for more practice.
- Applications of concepts in physics are discussed.

Dr. Savita Gahlaut is currently an associate professor at Deen Dayal Upadhyaya College, University of Delhi. She has a teaching experience of 25+ years at undergraduate level. She received her Ph.D in Physics from Delhi University. Her area of specialization is General Theory of Relativity and Cosmology.



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