

Wonderland of Numbers

Vol. I



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SULTAN CHAND & SONS

WONDERLAND of NUMBERS

(VOLUME – 1)

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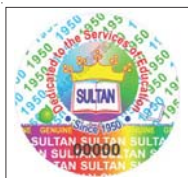
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Preface

To The First Edition

Numbers are used to collect information relating to crimes, military strength, population, wealth, etc. for devising military and fiscal policies. They are of fundamental importance in various diversified fields such as agriculture, industry, planning, economics, business management, finance, insurance, health services, banking, accountancy and auditing, and so on. Numbers are used by the government or business or management organisations in planning future programmes and formulating policy decisions. We use numbers all the time, right from getting up in the morning to going to bed at night. There are hardly any times when we are not surrounded by numbers. In today's world, numbers are indispensable in every nook and corner of our social, professional as well as personal and private life (telephones number, house number, pan card, aadhaar card, credit card, ATM card, etc.) It is rather impossible to think of any sphere of human activity where numbers do not creep in. Throughout history, numbers had a tremendous effect on our culture. *It seems that numbers rule the universe. Infact the numbers dictate our lives.*

*“Mathematics is the queen of sciences and number theory,
the queen of mathematics”* — Carl Friedrich Gauss

Shakuntala Devi, the great Indian mathematician and so called ‘Human Computer’, said:

*“Without mathematics there is nothing you can do.
Everything around you is mathematics.
Everything around you is numbers”.*

It is generally said that:

“Mathematics is a dull, dry and boring subject”,
and does not find favour with most of the students and the public at large.

(iv)

Preface

Our main objective in writing this book is to dispel these aspersions on mathematics. We believe that mathematics is very interesting and exciting subject. There is inherent beauty in mathematics, which only mathematics loving people can see, as the saying goes:

“Beauty lies in the eyes of the beholder”.

Prof. G.H. Hardy of Cambridge University said, *“There is no permanent place for ugly mathematics in the world”.*

It is said that *“if there is God, he is a great mathematician”.*

“Mathematics is the language with which God has written the Universe”.

— Galileo

This book is intended for the school children in particular and the general public at large. In fact this book is for anybody, really : young or old; rich or poor, it does not matter — even for those who are not so enthusiastic about mathematics. The only requirement is that one should be well versed in the four basic operations of arithmetic, viz., addition, subtraction, multiplication and division. Knowledge of mathematics upto high school level will be quite helpful and beyond that an added qualification.

Keeping in mind the varied requirements at different levels (of people w.r.t. knowledge of mathematics), an attempt has been made to start with the explanation of elementaries of a topic and then the complexities have been explained and solved in a lucid manner. A number of illustrations have been given and explained in a simple manner to enable the reader to have a better and thoughtful understanding of the basic concepts of theory and its applications. At many places, explanation, remarks/notes, scattered here and there throughout the book, have been given to widen the reader's horizon. Proper reasoning, logic and short proofs (not requiring rigorous mathematics) have been given to satisfy the inquisitive minds — for answering how and why.

The book gives an exciting, stimulating and adventurous voyage of exploration into the realm of numbers.

Numbers + Imagination = New Ideas

New Ideas + Imagination = Great Fun

“WELCOME TO THE WONDERLAND OF NUMBERS”

The book demonstrates that mathematics cannot be separated from meaning and that numbers have semantic content that makes them easier to handle.

It is hoped that the book will increase the readers’ appreciation for the inherent beauty that can be seen in mathematics and they will find mathematics more and more interesting and enjoyable.

The book contains Eight Chapters.

In Chapter 1, we have given a brief history of the development of various ‘*Numeration Systems*’ all over the world and the classification of the numbers into natural numbers, integers, rational and irrational numbers, real and complex numbers, etc. We have also discussed briefly various important results, theorems and terminology used in addition, subtraction, multiplication and division. Moreover, we have also given briefly the basic concepts of some mathematical topics taught in school.

Chapter 2 is devoted to the digits (0, 1, 2, ..., 9), together with many interesting and fascinating informations concerning them, *viz.*,

- (i) their mathematical properties,
- (ii) their common usage in day-do-day life,
- (iii) their religious, cultural and mythological significance,
- (iv) their use in various branches of science, *viz.*, physics, chemistry, biology, physiology, etc.,
- (v) their use in various diverse fields such as sports, movies, literature, history, etc.

Each digit reveals new excitement and delight.

In Chapter 3, ‘*Mathematical Tricks*’, we have given number of mathematical tricks based on the numbers and the four basic arithmetical operations, *viz.*, addition, subtraction, multiplication and division. After mastering these tricks you can amaze your friends, family members or the audience by some real ‘*number magic*’ and claim to possess some extra sensory powers of predictions and telepathy, and look like a mathematical wizard whose brain is a mega fast calculator.

In Chapter 4, '*Some Interesting Numbers*', we have described in detail some numbers which we thought to be more interesting than others because of their *unique, fascinating and exciting* properties. For example,

- (i) Amazing Number '1089',
- (ii) Revolving Number '142857',
- (iii) Fibonacci Numbers and Golden Ratio,
- (iv) Ramanujan Number '1729',
- (v) Beast Number '666', etc.,

to mention only a few.

In Chapter 5, '*Arithmetic Potpourri (Marvels in Arithmetic)*', we have discussed briefly a potpourri (or an assortment) of a number of beautiful flowers selected from the garden of mathematics in the form of some interesting topics, problem or results, primarily concerning numbers and their connection with various other areas of mathematics. These are bound to give you an insight into the inherent beauty in mathematics.

In Chapter 6, '*Square Numbers*', we have given the geometric interpretation of square numbers, their important properties, algebraic formulae and the beautiful patterns exhibited by them. We also discussed briefly the conditions under which any positive integer can be expressed as:

- (a) The difference of two squares of integers.
- (b) The sum of two squares of integers.

Chapter 7 is devoted to the study of '*Cube Numbers*' — their important properties, algebraic formulae and the beautiful patterns exhibited by them. Moreover Ramanujan's Number, Taxi-Cab Numbers and Cab-Taxi Numbers are also discussed in brief.

In Chapter 8, '*Srinivasa Ramanujan*', we have given a brief life history of the great Indian mathematical genius 'Srinivasa Ramanujan' and some of his important contributions to the world of mathematics.

We unreservedly acknowledge our deep debt of gratitude we owe to the numerous authors whose great and masterly works we have consulted during the preparation of the book.

Preface

(vii)

We take this opportunity to express our sincere gratitude to Prof. Kamal Nain Kapoor for his valuable help in the preparation of this book, and number of friends, and near and dear ones for their constant encouragement to write this book. A special thanks is due to Ms. Alisha Mahajan for her help in designing the cover page and icons for various chapters.

Last but not the least, we express our deep sense of gratitude to our publisher M/s Sultan Chand and Sons, Darya Ganj, New Delhi, particularly Shri Pratap Vaish, Dr. Shubhra and Smt. Shikha for their untiring efforts and unfailing courtesy and cooperation in bringing out this book in such an elegant form.

An attempt has been made to eliminate the mistakes and printing errors as far as possible. We shall be obliged if any such errors are brought to our notice. Valuable suggestions and criticism for the improvement of the book will be highly appreciated and duly incorporated in subsequent editions.

August 2021

S.C. GUPTA , ALKA MAHAJAN
VIKAS GUPTA , VISHAL GUPTA

Plato said, “*God is a Geometer*”. Jacobi changed it to, “*God is an arithmetician*”. Then came Kronecker and fashioned the memorable expression, “*God created the natural numbers, and all the rest is the work of man*”.

— *Felix Klein*

Contents

<i>Chapters</i>	<i>Pages</i>
1. Numbers — Meaning and Classification	1·1 – 1·61
Introduction	1·1
1·1 Number Theory	1·2
1·2 Short History of Numbers	1·5
1·2·1 Numbers and Numerals	1·7
1·3 Numeration Systems	1·8
1·3·1 Egyptian Numeration System	1·9
1·3·2 Babylonian Numeration System	1·9
1·3·3 The Roman System	1·11
1·3·4 Maya Numerals	1·13
1·3·5 Hindu-Arabic Numeration System	1·14
1·4 Key Elements of Number System	1·17
1·4·1 Conversion of Base (Remainder Algorithm)	1·19
1·5 Binary Number System	1·21
1·6 Classification of Numbers	1·23
1·6·1 Natural Numbers and Whole Numbers	1·23
1·6·2 Negative Numbers	1·23
1·6·3 Integers	1·24
1·6·4 Fraction	1·24
1·6·5 Rational Numbers	1·25
1·6·6 Decimals and Percentages	1·25
1·6·7 Irrational Numbers	1·26
1·6·8 Real Numbers	1·26
1·6·9 Complex Numbers	1·27
1·7 Definitions and Terminology	1·28
1·7·1 Terminology in Addition, Subtraction, Multiplication and Division	1·30
1·7·2 Factors	1·32
1·7·3 Even Numbers and Odd Numbers	1·33
1·7·4 Predecessor and Successor	1·33
1·7·5 Multiples	1·34

(x)**Contents**

1·8	Prime Numbers and Composite Numbers	1·34
1·8·1	Prime Factorisation	1·35
1·8·2	Prime Factorisation Theorem <i>OR</i> Fundamental Theorem of Arithmetic	1·36
1·9	Highest Common Factor (H.C.F.)	1·37
1·9·1	Co-prime Numbers	1·37
1·9·2	Computation of H.C.F. of Numbers	1·38
1·10	Lowest Common Multiple (L.C.M.)	1·41
1·10·1	Methods of Computing L.C.M.	1·41
1·11	Exponents and Laws of Indices	1·44
1·11·1	Laws of Exponents (or Indices)	1·45
1·12	Logarithms and their Laws	1·47
1·12·1	Laws of Logarithms	1·48
1·12·2	Uses of Logarithms	1·49
1·13	Square Root, Cube Root and n^{th} Root	1·49
1·13·1	Square Root	1·49
1·13·2	Cube-Root	1·50
1·13·3	n^{th} Root	1·51
1·14	Polynomial	1·51
1·14·1	Transcendental Number	1·52
1·14·2	Diophantine Equations	1·52
1·15	Linear and Quadratic Equations in One Variable	1·53
1·15·1	Linear Equation in One Variable	1·53
1·15·2	General Linear Equation in One Variable	1·54
1·15·3	Quadratic Equation in One Variable	1·54
1·15·4	Nature of Roots of (1·42)	1·56
1·16	Triangles	1·57
1·16·1	Kinds of Triangles (With Regard to their Sides)	1·57
1·16·2	Some Definitions	1·58
1·16·3	Properties of Triangles	1·59
1·16·4	Pythagoras Theorem	1·60
2.	Digits : (0, 1, 2, ..., 8, 9)	2·1 – 2·70
	Introduction	2·1
2·1	Digit – 1 (One)	2·1
2·2	Digit – 2 (Two)	2·6
2·3	Digit – 3 (Three)	2·10
2·4	Digit – 4 (Four)	2·16
2·5	Digit – 5 (Five)	2·20
2·6	Digit – 6 (Six)	2·23
2·7	Digit – 7 (Seven)	2·26
2·8	Digit – 8 (Eight)	2·37

Contents

(xi)

2· 9 Digit – 9 (Nine)	2·51
2·10 Digit – ‘0’ (Zero)	2·66
2·11 It’s All About Angles	2·69
3. Mathematical Tricks	3·1 – 3·52
Introduction	3·1
3·1 Magic Number 99	3·1
3·2 Magic Number 1089	3·3
3·3 Guessing the Correct Sum	3·5
3·4 Guessing Three-Digit Number	3·8
3·5 Guessing Four-Digit Number	3·10
3·6 Guessing the Two-Digit Number	3·12
3·7 Repeated Digits Numbers	3·13
3·8 Tricks Based on Number 9	3·13
3·8 (a) Trick With Three-Digit Number	3·14
3·9 Guessing the Hidden Digit of the Number	3·15
3·10 Guessing the Hidden (Suppressed) Digit (Continued)	3·17
3·10 (a) Guessing the Hidden Digit (Continued)	3·19
3·10 (b) Guessing the Hidden Digit (Continued)	3·20
3·11 Predicting the Remainder on Division By 9	3·21
3·12 Trick With Number 9	3·22
3·13 Trick With Number 8	3·23
3·14 Strange Matrix	3·24
3·15 A Strange and Surprising Sum	3·28
3·16 Trick Based on Fibonacci Type Sequence	3·29
3·17 Guessing Number of Brothers and Sisters of a Person	3·31
3·18 Guessing One’s Age	3·32
3·19 Guessing Date of Birth	3·33
3·20 Guessing One’s Age (A) and the Number of Coins (C) in the Pocket	3·34
3·21 Trick With A Dice	3·35
3·22 Trick With A Dice (Continued)	3·36
3·23 Trick With Two Dice	3·37
3·24 Prime Number Prediction	3·39
3·25 Prime Number Trick (Continued)	3·40
3·26 Recurring Number Magic	3·41
3·27 Magic Number 37	3·42
3·28 Magical Number 56	3·43
3·29 Trick With An Imaginary Number	3·44
3·30 Trick With Three Consecutive Odd Numbers	3·45
3·31 Trick With Four Consecutive Numbers	3·46

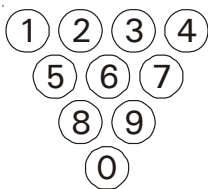
3-32	Magic Number 222	3-47
3-33	Magic Number 6666	3-48
3-34	Guessing the Number	3-50
3-35	Trick Based on a Peculiar Series	3-51
4.	Some Interesting Numbers	4-1 – 4-76
	Introduction	4-1
4-1	Amazing Number 1089	4-2
4-2	Ramanujan's Number — 1729	4-5
4-3	Kaprekar Constant — 6174	4-7
	4-3-1 Kernel 495 for 3-Digit Numbers	4-10
4-4	The Revolving Number — 142857	4-11
4-5	Beast Number 666	4-17
4-6	Curious Number 153	4-20
4-7	Palindromic Numbers	4-24
	4-7-1 How to Generate Palindromes ?	4-25
	4-7-2 Stubborn 196	4-26
4-8	Fibonacci Numbers	4-27
	4-8-1 Properties of Fibonacci Numbers	4-28
4-9	Golden Ratio (G.R.)	4-34
	4-9-1 Relationship Between Golden Ratio and Fibonacci Numbers	4-37
	4-9-2 Golden Ratio as Continued Square Root	4-38
	4-9-3 Golden Ratio as Continued Fraction	4-38
	4-9-4 Golden Ratio in Nature	4-39
	4-9-5 "Golden Ratio" in Human Body	4-39
	4-9-6 Golden Ratio and Perfect Face Or Mathematics Behind Beautiful Face	4-40
4-10	Golden Rectangle	4-41
	4-10-1 Construction of Golden Rectangle	4-41
	4-10-2 Golden Spiral	4-42
4-11	Pell's Numbers	4-43
	4-11-1 Pell's Numbers and the Rational Approximations to the Square Root of 2	4-44
	4-11-2 Properties of Pell's Numbers	4-45
	4-11-3 Pell's Numbers and Square Triangular Numbers	4-45
	4-11-4 Pell's Equation	4-46
4-12	Silver Ratio	4-48
	4-12-1 Silver Ratio and Pell's Numbers	4-49
	4-12-2 Silver Ratio in Terms of Continued Fraction	4-49
4-13	Silver Rectangle	4-50

4-13-1	Geometric Representation of Silver Ratio and Silver Rectangle	4-51
4-13-2	Silver Spiral	4-53
4-14	Numbers 11 and 110	4-53
4-15	Number '13'	4-55
4-16	Number 23	4-67
4-17	Some More 'Interesting Numbers'	4-71
5.	Arithmetic Potpourri (Marvels In Arithmetic)	5-1 – 5-56
	Introduction	5-1
5-1	Winners	5-1
5-2	$(3n + 1)$ Problem or Collatz Conjecture	5-3
5-3	Camel Problem	5-5
5-4	Fibonacci Rabbit Mating Problem	5-7
5-5	The Steinhaus Cycle	5-8
5-6	Interesting History of 1752 (April Fool Day)	5-10
5-7	Multiplication By Doubling and Halving	5-11
5-8	Cows and Milk Problem	5-13
5-9	Mathematical Study of Secret of Success	5-15
5-10	Playing Cards and Calender	5-17
5-11	Mathematics and Music	5-19
5-12	Calender Day	5-21
5-13	Anniversaries	5-23
5-14	Mnemonics	5-25
5-15	Beauty in Equations	5-27
5-16	Interesting Fractions	5-32
5-17	Amazing Fractions (Recurring Decimals) — I	5-33
5-18	Amazing Fractions (Recurring Decimals) — II	5-36
5-19	Number "2 777 777 888 888 99"	5-39
5-20	Some Interesting Number Facts	5-42
5-21	Multiplication Table of '9' on Your Fingers	5-46
5-22	Multiplication Tables of Two and Three Digited Numbers	5-47
5-22-1	How to Write Multiplication Tables of Two Digited Numbers?	5-47
5-22-2	How to Write Tables of Three-digitd Numbers	5-49
5-23	'Digital Root' or 'Casting Out Nines'	5-50
6.	Square Numbers	6-1 – 6-40
	Introduction	6-1
6-1	Properties Of Square Numbers	6-1
6-1-1	Some Algebraic Results on Square Numbers	6-5

6-2	Patterns Exhibited By Square Numbers	6-8
6-3	Geometric Interpretation of Square Numbers	6-11
6-4	Square Numbers Using Each Digit 1 to 9	
	Exactly Once	6-13
	6-4-1 “Square Numbers”, Using Each Digit 0 To 9	
	Exactly Once	6-15
	6-4-2 Some Interesting Square Numbers	6-16
6-5	Integers Expressible as Difference of Two Squares	6-16
	6-5-1 Difference of Two Consecutive Squares	6-17
	6-5-2 Difference of Two Consecutive Odd Squares	6-17
	6-5-3 Difference of Two Consecutive Even Squares	6-18
6-6	Integers Expressible as Sum of Two Squares	6-19
	6-6-1 Sum of Two Consecutive Odd Squares	6-20
	6-6-2 Sum of Two Even Squares	6-20
	6-6-3 Sum of Two Consecutive Squares	6-21
	6-6-4 Fermat’s Theorem on Sum of Two Squares	6-22
	6-6-5 Sum of Two Squares Theorem	
	(Prime Factorisation Theorem)	6-24
	6-6-6 Some More Results on Sum of Two Squares	6-26
	6-6-7 Diophantus Identity on Sum of Two Squares	6-27
	6-6-8 Integers Expressible as the Sum of	
	Two Integer Squares in Two Different Ways	6-27
	6-6-9 Sum of Two Squares in Three Different Ways	6-30
6-7	Legendre’s Three Square Theorem	6-31
	6-7-1 Gauss Theorem on Sum of Three Squares	6-32
6-8	Sum of Four Squares of Integers	6-34
	6-8-1 Euler’s Four Square Identity	6-35
	6-8-2 Lagrange’s Four Square Theorem	6-35
6-9	Sum of Five Squares	6-37
6-10	Sum of ‘ k ’ Squares	6-38
6-11	An Amazing Circle of Square Numbers	6-40
7.	Cube Numbers	7-1 – 7-14
	Introduction	7-1
	7-1 Properties of Cube Numbers	7-2
	7-2 Ramanujan-Hardy Number — 1729	7-7
	7-2-1 Ramanujan’s Number and Prime Numbers	7-7
	7-2-2 Ramanujan’s Numbers	7-7
	7-2-3 Ramanujan’s Triplets	7-8
	7-3 Taxi-Cab Numbers	7-9
	7-4 Cab-Taxi Numbers	7-10

7-5	Fermat's Last Theorem	7-10
7-6	Cube as a Sum of Three Consecutive Cubes	7-11
7-7	Some Algebraic Results on Cube Numbers	7-13

8.	Srinivas Aiyenger Ramanujan (FRS)	
	and His Contributions to Mathematics	8-1 – 8-26
	Introduction	8-1
8-1	Ramanujan – Hardy Number — ‘1729’	8-8
8-2	Ramanujan's Numbers and Ramanujan's Triplets	8-8
8-3	Taxi-Cab Numbers and Cab-Taxi Numbers	8-9
8-4	Ramanujan's Pi (π) Formulae	8-9
8-5	Ramanujan's Constant	8-10
8-6	Ramanujan's Conjecture	8-11
8-7	Ramanujan's Sum	8-12
8-8	Ramanujan's Famous Infinite Root	8-14
8-9	Ramanujan's Prime Numbers	8-15
	8-9-1 Prime-counting Function	8-15
	8-9-2 Gauss's Approximation for $\pi(x)$	8-16
	8-9-3 Ramanujan's Prime Numbers	8-17
8-10	Partitioning of Numbers	8-17
	8-10-1 Formula for $p(n)$	8-18
	8-10-2 Approximation for $p(n)$	8-19
	8-10-3 Uses	8-19
8-11	Highly Composite Numbers	8-20
8-12	Ramanujan's Most Beautiful Formula	8-23
8-13	Ramanujan and Fermat's Last Theorem	8-23
8-14	Ramanujan's Conjecture and Ramanujan – Nagell Equation	8-23
8-15	Landau – Ramanujan Constant	8-24
8-16	Ramanujan's Magic Square	8-25



Digits : (0, 1, 2, ..., 8, 9)

INTRODUCTION

In the Hindu-Arabic numerals or Indo-Arabic numerals, there are ten digits :

$$0, 1, 2, 3, 4, 5, 6, 7, 8, 9;$$

which are the most common symbolic representation of the numbers in the world today.

In a given number system, if the base is an integer, then the number of digits required is always equal to the absolute value of the base. The decimal system (base 10), has ten digits, *viz.*, 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9; whereas the binary system (base 2), has two digits (0, 1).

(0, 1, 2, ..., 8, 9) are both a number and the numerical digit to represent that number. In this Chapter, you will come to know all the things, in brief, you wanted to know about these digits (numbers) :

- (i) Their important mathematical properties.
- (ii) Their common usage in day-to-day life.
- (iii) Their religious, cultural and mythological significance all over the world.
- (iv) Their use in various branches of science, *viz.*, physics, chemistry, biology, physiology, astronomy, etc.
- (v) Their use in various diverse fields such as sports, music, movies, literature, history, etc.



2·1

DIGIT – 1 (ONE)

Mathematical Properties

- ◆ ‘1’ is the first natural number that follows zero and precedes 2.

(iii) Terror strike on *Mumbai Hotel* on 26th, ($2 + 6 = 8$), November 2008.

(iv) Kashmir earthquake (Pakistan) on 08th October, 2005.



2-9

DIGIT – 9 (NINE)

◆ ‘Number 9’ possesses many interesting, rather outstanding properties which have earned it the name :

‘*Amazing number 9*’ or ‘*Magic number 9*’ or ‘*Fascinating number 9*’ or ‘*mind blogging number 9*’. The speciality of the number 9 can never be exaggerated.

Mathematical Properties

- ◆ 9 is a natural number following 8 and preceding 10.
- ◆ 9 is built up of 3’s. $9 = 3 + 3 + 3 = 3 \times 3$.
- ◆ 9 is the highest single digit number in base 10.
- ◆ 9 is a composite number, its proper (aliquot) divisors — (all the divisors excluding the number itself), being 1 and 3.
- ◆ 9 is the *least* number, the sum of whose proper divisors is a square. $[1 + 3 = 4 = 2^2]$.
- ◆ 9 is the first composite odd number as well as the first composite lucky number.
- ◆ In binary notation, 9 can be expressed as : $9 = (1001)_2$.
- ◆ A polygon of 9 sides is called *nonagon* or *enneagon*.
- ◆ 9 is a square number; $[9 = 3^2]$.
- ◆ It is the *only* square which can be expressed as the sum of two consecutive cubes. $[9 = 3^2 = 1^3 + 2^3]$.
- ◆ Each and every *two-digit* number that ends in 9 is the sum of the products of the two digits and the sum of the two digits. For example :

$$29 = (2 \times 9) + (2 + 9); \quad 89 = (8 \times 9) + (8 + 9); \text{ and so on.}$$
- ◆

$$9 = 5 + 4 = 5^2 - 4^2 = 1^3 + 2^3$$

$$= 0^4 + 1^3 + 2^2 + 3^1 + 4^0$$

$$= 1! + 2! + 3! \quad (\text{Sum of consecutive factorials})$$

Then the above steps give :

$$\begin{array}{cccccc}
 [& 5 & (2B + 3) & + & S &] - 15 \\
 \downarrow & & \downarrow & \downarrow & \downarrow & \downarrow \\
 III & & I & II & IV & V \\
 & & & & = 10B + S \text{ (On simplification)} \\
 & & & & = B & S \\
 & & & & \downarrow & \downarrow \\
 & & & & \text{Ten's} & \text{Unit's} \\
 & & & & \text{digit} & \text{digit}
 \end{array}$$

In the above formula, take $B = 0$ (or $S = 0$) if a person has no brother (or sister).

Another modified version of the formula is given below :

$$[2(5B + 4) + S] - 8 = 10B + S = BS$$

See, if you can find some different versions by slight manipulations in the above formula !

$$\text{Try } \{ [5(B + 1) + 3] \times 2 + S \} - 16$$

You need to bring 10 as multiple of B . For example another version may be

$$[10(B + 2) + S] - 20 = BS$$



Trick No. 3-18

GUESSING ONE'S AGE

You can tell the age of persons by asking them to do the following calculations :

Ask them to multiply their age successively by 7, 13 and 37 and handover the resulting product to you.

What you have to do is simply to multiply this result by 3. You will get the persons' age repeated three times, from which you can tell their age.

Illustration 1

$$\begin{array}{l}
 \text{Age} = 64 \text{ years} \\
 64 \times 7 = 448 \\
 448 \times 13 = 5824 \\
 5824 \times 37 = 215488 \\
 215488 \times 3 = 64 \ 64 \ 64 \\
 \text{Age} = 64 \text{ years}
 \end{array}$$

Illustration 2

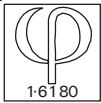
$$\begin{array}{l}
 \text{Age} = 28 \text{ years} \\
 28 \times 7 = 196 \\
 196 \times 13 = 2548 \\
 2548 \times 37 = 94276 \\
 94276 \times 3 = 28 \ 28 \ 28 \\
 \text{Age} = 28 \text{ years}
 \end{array}$$

Writing the letters of the alphabet from the above table for the digits in (*), we get in order :

LOO FLI RPA

Writing this in reverse order (from right to left) we get :

APRIL FOOL



4·2

RAMANUJAN'S NUMBER — 1729

1729 is popularly known as Ramanujan's number, after the famous story of the British mathematician Professor G.H. Hardy (Fellow of Trinity College, Cambridge), regarding his visit to the hospital to see the Indian mathematician Srinivas Iyenger Ramanujan when he was ill. In Hardy's words :

"I remember once going to see him when he was ill at Putney. I had ridden a taxi cab number 1729 and remarked that the number seemed to me rather a dull one, and that, I hoped it was not an unfavourable omen".

Ramanujan had marvellous mental and intuitive powers. On hearing Hardy's remarks, Ramanujan's face lit up with excitement and he told Professor Hardy that 1729 was a very special and interesting number with the following property.

"1729 is the smallest number which can be expressed as the sum of two cubes in two different ways".

$$1729 = 1000 + 729 = 10^3 + 9^3$$

$$1729 = 1 + 1728 = 1^3 + 12^3$$

Note. In fact, in the above quotation, we should use the word 'positive cubes' instead of 'cubes', because if negative perfect cubes, i.e., cube of a negative quantity is allowed, then the smallest solution is 91, which is a factor of 1729.

$$91 = 6^3 + (-5)^3 = 4^4 + 3^3$$

Some Mathematical Properties of 1729

1. Factors of 1729 are :

$$1729 = 1 \times 7 \times 13 \times 19.$$

Its prime factors : (7, 13, 19) form a series in Arithmetic Progression with first term = $a = 7$ and difference (d) = $13 - 7 = 19 - 13 = 6$.

7		1729
13		247
		19

Each term in the expansion of $1/7$ is double the previous term shifted two places to the right [See (*)]. This basically accounts for the property 6.

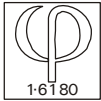
3. Are there any other numbers with the amazing property 1? The answer is Yes ! In fact, there are infinitely many. All such numbers arise by the recurring decimal presentations of the numbers of the form $(1/p)$, where p is a *cyclic prime*.

First few cyclic primes are 7, 17, 19, 23, 29, 47, 59, 61, etc. The recurring numbers (with the Property 1) corresponding to 17 and 19 are given below.

$$\frac{1}{17} = 0.05882\ 35294\ 117647 \quad (16 \text{ digit recurring number})$$

$$\frac{1}{19} = 0.0526\ 3157\ 8947\ 368421 \quad (18 \text{ digit recurring number}).$$

It is necessary to add (prefix) 0 to these numbers for the Property 1.



4·5

BEAST NUMBER 666

The number 666 is called the *Beast Number*. The nefarious connotations of 666 are rooted in Bible. The first book of the *New Testament* in *Revelation* which tells of an *end of all time* when a great war will be fought between good and evil, and Good will triumph. In *Revelation*, “*the beast*”, also called the *Antichrist*, *personifies evil*.

Chapter 13, verses 16 and 17 of Revelation say that the followers of the beast will have the mark or the name or the number of the beast on their right hands or foreheads. *Verse 18*, introduces the number 666 itself. “Let him that hath understanding count the number of the beast, for it is the number of a man; and his number is six hundred three score and six”.

During the Second World War (1939-1945), Adolf Hitler of Germany was bestowed the title of the Beast. If we identify *A, B, C, D, ...,* with numerals as follows :

$A = 100, B = 101, C = 102, \dots, X = 123, Y = 124$ and $Z = 125$; then we get

$$H = 107$$

$$I = 108$$

$$T = 119$$

$$L = 111$$

$$E = 104$$

$$R = 117$$

$$\text{HITLER} = 107 + 108 + 119 + 111 + 104 + 117 = \underline{\underline{666}}$$

$$= \frac{1}{7} [96] = 13 \frac{5}{7}$$

Remainder = 5 \Rightarrow Day on 16/01/1756 was *Friday*.

Illustration 3. Find the day on 22 June, 1603.

Solution. Using the formula in (i) or (*), we get

$$\begin{aligned} \frac{S}{7} &= \frac{1}{7} [22 + 03 + 0 + 4 + 6]; \quad \left[\text{Quotient in } \frac{03}{04} \text{ is zero.} \right] \\ &= \frac{1}{7} (35) = 5 \end{aligned}$$

Remainder = 0 \Rightarrow Day on 22 June 1603 was *Sunday*.

Illustration 4. D.O.B. of a person is 9 February, 1979. What was the day of his birth?

Solution. Using the formula in (*), we get

$$\frac{S}{7} = \frac{1}{7} [9 + 79 + 19 + 3 + 0] = \frac{110}{7} = 15 \frac{5}{7}$$

Remainder = 5 \Rightarrow Day of birth = *Friday*



5-13

ANNIVERSARIES

Anniversary is the *date* on which an *event* took place in a previous year. It seems to have been derived from the Latin word : “*Anniversarius*” — a returning year.

$$\begin{array}{c} [\text{Anniversarius} = \text{Annus} + \text{Versus}] \\ \quad \quad \quad \downarrow \quad \quad \downarrow \\ \quad \quad \quad (\text{Year}) \quad (\text{Turning}) \end{array}$$

The event may be the date on which (in a previous year),

- (i) a person was born,
- (ii) a couple was married,
- (iii) an important event was inaugurated or launched,
- (iv) some historical event like ‘independence of a country/nation’ or ‘adoption of constitution’, and so on.

One year later would be the *first* anniversary of that event.

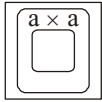
Most common anniversaries for a layman are :

- (i) The ‘*Birthday Anniversary*’, and
- (ii) The ‘*Wedding Anniversary*’.

From Table 6-12, we observe that certain numbers are expressible as sum of squares of non-zero integers in more than one way.

For example :

- (i) 17 is expressible as sum of 2, 3, 5 and 6 integer squares.
- (ii) 21 is expressible as sum of 3, 4, 5, 6 integer squares,
and so on, as given in the illustrations.



6-11

AN AMAZING CIRCLE OF SQUARE NUMBERS

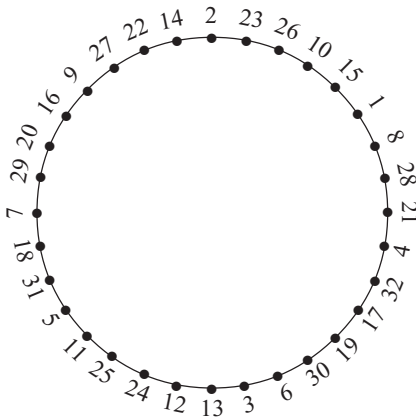


Fig. 6-3

This is an amazing circle in which the 32 numbers from 1 to 32, each number occurring only once, have been so arranged that the sum of any two adjacent numbers is a perfect squares, *e.g.*,

$$31 + 18 = 49 = 7^2$$

$$1 + 15 = 16 = 4^2$$

$$31 + 5 = 36 = 6^2$$

$$1 + 8 = 9 = 3^2$$

$$26 + 23 = 49 = 7^2$$

$$13 + 3 = 16 = 4^2$$

$$26 + 10 = 36 = 6^2$$

$$13 + 12 = 25 = 5^2$$

Isn't it really fascinating?

Remark. If negative numbers are also allowed, then some of *Ramanujan's triplets* are :

$$4\ 104 = 2^3 + 16^3 = 9^3 + 15^3 = (-12)^3 + 18^3$$

$$3\ 242\ 197 = [(141, 76), (138, 85), (-171, 202)].$$

Sum of Two Cubes (Positive and Negative) in Four Different Ways

$$42\ 549\ 416 = 348^3 + 74^3$$

$$= 282^3 + 272^3$$

$$= (-475)^3 + 531^3$$

$$= (-2662)^3 + 2664^3$$

Sum of Two Cubes (Positive and Negative) in Five Different Ways

$$1\ 148\ 834\ 232 = 222^3 + 1044^3$$

$$= 718^3 + 920^3$$

$$= 816^3 + 646^3$$

$$= (-1425)^3 + 1593^3$$

$$= (-7986)^3 + 7992^3$$



7-3

TAXI-CAB NUMBERS

The smallest numbers which can be expressed as the sum of two positive cubes in n different ways are called Taxi-cab (n) numbers.

In 1938, Prof. G.H. Hardy (of Cambridge University) and E.M. Wright proved that such numbers exist for all positive integer values of $n \geq 1$, but finding such numbers is quite tedious. However, their proof is easily converted into a computer programme to generate such numbers.

The first few known *Taxi-cab numbers* are given below :

Taxi-cab (1) = 2 = 1³ + 1³

Taxi-cab (2) = Ramanujan's Number = 1729

$$= (1^3 + 12^3, 10^3 + 9^3).$$

$$= [(1, 12), (10, 9)]$$

Taxi-cab (3) = 87 539 319

$$= [(228, 423); (167, 436); (255, 414)]$$

Taxi-cab (4) = 6 963 472 309 248

$$= \left[(13\ 322, 16\ 630); (10\ 200, 18\ 072); (5\ 436, 18\ 948); (2421, 19\ 083) \right]$$



8-12

RAMANUJAN'S MOST BEAUTIFUL FORMULA

$$\left[1 + \frac{1}{1 \cdot 3} + \frac{1}{1 \cdot 3 \cdot 5} + \frac{1}{1 \cdot 3 \cdot 5 \cdot 7} + \dots \right] + \left[\frac{1}{1 + \frac{1}{1 + \frac{2}{1 + \frac{3}{1 + \frac{4}{\dots}}}}} \right] = \sqrt{\frac{\pi e}{2}} \quad \dots (8-29)$$

Note that, neither the infinite series nor the continued fraction on the left hand side are related to π or e .



8-13

RAMANUJAN AND FERMAT'S LAST THEOREM

Fermat's Last Theorem states :

The Diophantine equation :

$$x^n + y^n = z^n, \quad \dots (8-30)$$

has no integer solution for $n > 2$.

Ramanujan almost provided a counter-example to it.

He obtained the triplet (x, y, z) such that

$$(x = 65601, \quad y = 67402, \quad z = 83802).$$

$$\Rightarrow x^3 = 65601^3 = 282, 313, 326, 276, 801$$

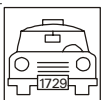
$$y^3 = 67402^3 = 306, 209, 281, 368, 808$$

$$x^3 + y^3 = 588, 522, 607, 645, 609$$

$$z^3 = 83802^3 = 588, 522, 607, 645, 608$$

$$\therefore \boxed{x^3 + y^3 = z^3 + 1.} \quad \dots (8-31)$$

(Just 1 short of a cube)



8-14

**RAMANUJAN'S CONJECTURE AND
RAMANUJAN - NAGELL EQUATION**

Consider the exponential Diophantine equation

About the Book

The book, a result of extensive research on numbers, is intended for the general public – even for those who are not so enthusiastic about mathematics. It conveys the message that mathematics is a very exciting and interesting subject. Numbers possess some amazing properties which will not only amaze you but also provide thrill, excitement and fun, and keep you spell bound. The mathematical ideas in the book provide hours, days and months, if not years of “entertaining numbers”. Moreover, it will also result in increasing the general knowledge of the readers. The book takes you through a thrilling, stimulating and adventurous voyage of exploration into the realm of numbers.

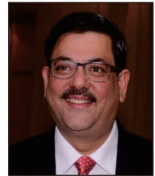
Numbers + imagination = New ideas,
New ideas + imagination = Great fun,
“Welcome to the Wonderland of Numbers”

S.C. Gupta, was an eminent and dedicated faculty member of Hindu College, University of Delhi since 1963 and retired in 2002 as Associate Professor in Statistics. His teaching career spanned over four decades. A well established author of international repute, he has written books on Mathematical Statistics, Fundamentals of Statistics, Applied Statistics, Business Statistics, Engineering Mathematics and Matrices. For the last few years, he had developed special passion for numbers and researched them extensively, resulting in this book for the general public.



Alka Mahajan, a Statistics major from Delhi University, has 15+ years of experience of working as a mathematics teacher in international schools, both in India and England. She has done Montessori Teacher Training from MMI, London and is certified Primary Years Programme (PYP) International Baccalaureate from Cambridge University. She has an unparalleled ability to explain complicated concepts in an easy understandable manner, making this book engaging and fun for learners of all ages.

Vikas Gupta, a technology professional, IT leader and an author has a brilliant academic record; Master of Computer Applications and B.Sc. Hons. in Mathematics from Delhi University. He has done Executive MBA from Cass Business School, University of London and is currently working as Business Head with Motherson Infotech and Solutions UK Ltd. (MIND) and lives in Reading (Berkshire), UK. He is also the co-author of the books: “Problems and Solutions in Mathematical Statistics” and “Mathematics Digest X”.



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