S.C. Gupta • V.K. Kapoor

FUNDAMENTALS OF MATHEMATICAL STATISTICS

Twelfth Edition

SULTAN CHAND & SONS

Fundamentals of Mathematical Statistics

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Preface to the Twelfth Edition

Knowledge updating is a never-ending process and so should be the revision of an effective textbook. The book originally written **fifty years ago** has, during the intervening period, been revised and reprinted several times. The authors have, however, been thinking, for the last few years that the book needed not only a thorough revision but rather a substantial rewriting. They now take great pleasure in presenting to the readers the **twelfth,** thoroughly revised and enlarged, **Golden Jubilee edition** of the book. The subject-matter in the entire book has been re-written in the light of numerous criticisms and suggestions received from the users of the earlier editions in India and abroad. The basis of this revision has been the emergence of new literature on the subject, the constructive feedback from students and teaching fraternity, as well as those changes that have been made in the syllabi and/or the pattern of examination papers of numerous universities.

Brief recall of some salient features of the present edition:

- The entire text has been restructured, rewritten and updated to cater to the revised syllabi of almost all universities, and various other professional examinations.
- During the course of rewriting, it has been specially borne in mind to retain all the basic features of the previous editions especially the simplicity of presentation, lucidity of style and analytical approach which have been appreciated by the readers throughout the country.
- A conscious effort has been made to make the presentation of all topics discussed very simple and to develop the subject-matter in such step-by-step manner that the book can serve as self-study text for the students.
- A number of typical problems have been added as solved examples in each chapter. These will enable the reader to have a better and thoughtful understanding of the basic concepts of the theory and its various applications.
- Several new topics have been added at appropriate places in various chapters to make the treatment of subject matter more exhaustive and up-to-date.

Some prominent additions are given below:

§ 6·5·1	:	Page 6·11	:	Variance of Degenerate Random Variable
§ 6·5·2	:	Page 6·12	:	Approximate Expression for Expectation and Variance
Remark 6	:	Page 6·40	:	A·M ≥ G.M., Inequality
§ 6·7·4	:	Page 6·42	:	Lyapounov's Inequality
§ 6·7·5	:	Page 6·42	:	C_r Inequality
§ 6·7·6	:	Page 6·44	:	Holder's Inequality
§ 6·7·7	:	Page 6·45	:	Minkowski's Inequality

§ 6·9·1	:	Page 6·48	:	Double Expectation Rule or Double-E Rule
§ 6·9·2	:	Page 6·50	:	Conditional Variance
§ 6·9·3	:	Page 6·50	:	Wald's Theorem on Mean and Variance of a Random Sum
§ 6·9·4	:	Page 6·51	:	Multistage Rule for Expectation (Multi-stage-E Rule)
§ 7·4	:	Page 7·25 to Page 7·27	:	Theorem 7·1 to Theorem 7·5
§ 7·5·2	:	Page 7·38	:	Generalisation of Chebychev's Inequality
§ 7·5·3	:	Page 7·39	:	Frechet Inequality
§ 7·5·4	:	Page 7·39	:	Bernstein's Inequality
§ 7·6	:	Page 7·40	:	Modes of Convergence of Sequence of Random Variables
§ 7·6·1	:	Page 7·40 to Page 7·46	:	Convergence in Probability Theorems 7·11 to Theorem 7·16
§ 7·6·2	:	Page 7·46	:	Almost Sure Convergence or Convergence With Probability One
	:	Page 7·47	:	Theorem 7·17
§ 7·6·3	:	Page 7·48	:	Convergence in *r*th-Mean Theorems 7·18 and 7·19
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§ 7·6·5	:	Page 7·55	:	Relationship Between Various Modes of Convergence
§ 7·8	:	Page 7·72	:	Strong Law of Large Numbers
§ 7·8·1	:	Page 7·72	:	Kolmogorov's Inequality
§ 7·8·2	:	Page 7·75	:	Strong Law of Large Numbers (S.L.L.N.)
§ 7·8·3	:	Page 7·75	:	Kolmogorov's First Theorem on S.L.L.N.
§ 7·8·4	:	Page 7·76	:	Kolmogorov's Second Theorem on S.L.L.N.
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§ 7·10·3	:	Page 7·85	:	Factorial Moment Generating Function
§ 8·4·3 (i)	:	Page 8·13	:	Factorial Moment Generating Function (Binomial Distribution)
§ 8·6·3	:	Page 8·54	:	Additive Property of Negative Binomial Distribution
§ 8·9·2	:	Page 8·70	:	Applications of Multinomial Distribution
Remarks 1, 2, 3	:	Page 9·67	:	On Cauchy Distribution
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§ 9·13·1	:	Page 9·69	:	Moments
§ 9·13·2	:	Page 9·69	:	Uses
§ 9·13·3	:	Page 9·70	:	Erlang Family of Distributions

§ 9·14	:	Page 9·71	:	Pareto Distribution
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§ 20·2·1	:	Page 20·18	:	MVND p.d.f.
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Problem sets at the end of each chapter are substantially reorganised. Many new problems are included at the end of each chapter. Repetition of problems of the same type (more than what is necessary) has been avoided. Further, the problems have been carefully arranged and properly graded according to the level of difficulty.

- Solved examples and unsolved problems have been drawn from the latest examination papers of various universities, professional examinations, etc.

- An attempt has been made to rectify the errors in the previous edition.
- The present edition incorporates modern viewpoints. In fact with the addition of new topics, rewriting and revision of many others and restructuring of problem sets, altogether a new book, covering the revised syllabi of almost all the universities, is being presented to the reader. It is earnestly hoped that, in the new form, the book will prove of much greater utility to the students as well as teachers of the subject and research scholars.

A list of all the individuals to whom we owe gratitude would exhaust readers' patience and intimidate our memory. A less personal but still sincere way is to extend thanks to the students who endured, to the fellow colleagues and associates who contributed, to an environment that inspired, to the authors that permitted, to the reviewers that critiqued, and to our family that waited. To these donors, we dedicate whatever is worthwhile in the book; the shortcomings and vagaries are, however, our responsibility.

We express our deep sense of gratitude to our Publishers, SULTAN CHAND & SONS for their untiring efforts, unfailing courtesy, and co-operation in bringing out the book, in such an elegant form.

Suggestions and constructive criticism for further improvement of the book as well as intimation of errors and misprints will be most gratefully acknowledged. And of course, given complexity of the subject-matter, we crave for readers' indulgence for any omission, error or discrepancy that might have crept in the text despite due care and caution on our part.

July 31, 2020 **S.C. GUPTA • V.K. KAPOOR**

Preface to the First Edition

Although there are a large number of books available covering various aspects in the field of Mathematical Statistics, there is no comprehensive book dealing with the various topics on Mathematical Statistics for the students. The present book is a modest though determined bid to meet the requirements of the students of Mathematical Statistics at Degree, Honours and Post-graduate levels. The book will also be found of use by the students preparing for various competitive examinations. While writing this book our goal has been to present a clear, interesting, systematic and thoroughly teachable treatment of Mathematical Statistics and to provide a textbook which should not only serve as an introduction to the study of Mathematical Statistics but also carry the student on to such a level that he can read with profit the numerous special monographs which are available on the subject. In any branch of Mathematics, it is certainly the teacher who holds the key to successful learning. Our aim in writing this book has been simply to assist the teacher in conveying to the students more effectively a thorough understanding of Mathematical Statistics.

The book contains sixteen chapters. The first chapter is devoted to a concise and logical development of the subject. The second and third chapters deal with the frequency distributions, and measures of average and "dispersion. Mathematical treatment has been given to the proofs of various articles included in these chapters in a very logical and simple manner. The theory of probability which has been developed by the application of the set theory has been discussed quite in detail. A large number of theorems have been deduced using the simple tools of set theory. The simple applications of probability are also given. The chapters on mathematical expectation and theoretical distributions (discrete as well as continuous) have been written keeping the latest ideas in mind. A new treatment has been given to the chapters on correlation, regression and bivariate normal distribution using the concepts of mathematical expectation. The thirteenth and fourteenth chapters deal mainly with the various sampling distributions and the various tests of significance which can be derived from them. In chapter 15, we have discussed concisely statistical inference (estimation and testing of hypothesis). Abundant material is given in the chapter on finite differences and numerical integration. The whole of the relevant theory is arranged in the form of serialised articles which are concise and to the point without being insufficient. The more difficult sections will, in general, be found towards the end of each chapter. We have tried our best to present the subject so as to be within the easy grasp of students with varying degrees of intellectual attainment.

Due care has been taken of the examination needs of the students and, wherever possible, indication of the year, when the articles and problems were set in the

examination has been given. While writing this text, we have gone through the syllabi and examination papers of almost all Indian universities where the subject is taught so as to make it as comprehensive as possible. Each chapter contains a large number of carefully graded worked problems mostly drawn from university papers with a view to acquainting the student with the typical questions pertaining to each topic. Furthermore, to assist the student to gain proficiency in the subject, a large number of properly graded problems, mainly drawn from examination papers of various universities, are given at the end of each chapter. The questions and problems given at the end of each chapter, usually require for their solution, a thoughtful use of concepts. During the preparation of the text we have gone through a vast body of literature available on the subject, a list of which is given at the end of the book. It is expected that the bibliography given at the end of the book will considerably help those who want to make a detailed study of the subject.

The lucidity of style and simplicity of expression have been our twin objects to remove the awe which is usually associated with most mathematical and statistical textbooks.

While every effort has been made to avoid printing and other mistakes, we crave for the indulgence of the readers for the errors that might have inadvertently crept in. We shall consider our efforts amply rewarded if those for whom the book is intended are benefited by it. Suggestions for the improvement of the book will be highly appreciated and will be duly incorporated.

September 7, 1970 **S.C. GUPTA • V.K. KAPOOR**

We dedicate this book to

Our respected teacher Prof. Harish Chander Gupta who initiated the teaching of Mathematical Statistics at Post-graduate level at the University of Delhi in the year 1958.

... And with special thanks to our wives, who have cheerfully endured the hardships of an author's life and are always very supportive. We are proud of them.

... And most importantly, we must also put on record our gratitude to two great esteemed institutions, Hindu College and Shriram College of Commerce , (the constituent colleges of University of Delhi), for all that we learnt as a teacher for more than four decades from these premier institutions of South-east Asia

... Last but not the least, thanks to almighty God for giving us the talent to write and share the fruits of this wonderful gift with students.

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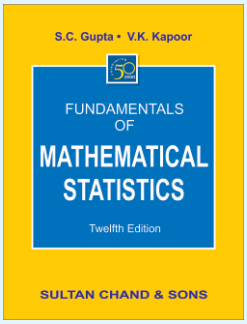

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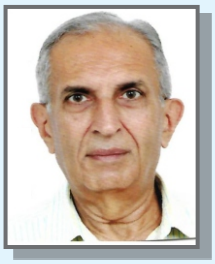

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