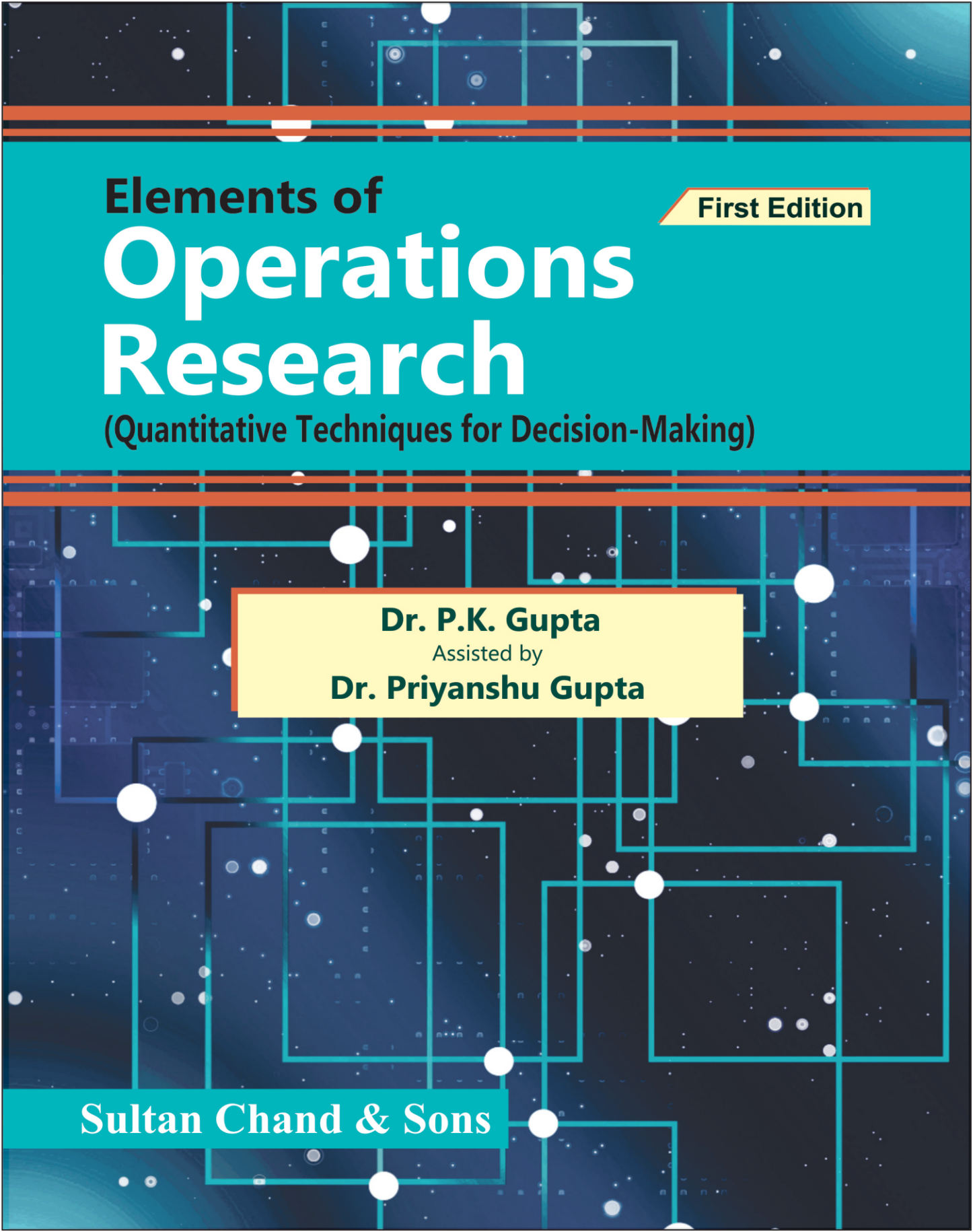




Elements of Operations Research

First Edition

(Quantitative Techniques for Decision-Making)

Dr. P.K. Gupta

Assisted by

Dr. Priyanshu Gupta

Sultan Chand & Sons

First Edition

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PREFACE

It has been widely acknowledged the importance of Operations Research/Quantitative Techniques in decision-making. The need for a thorough understanding of various Operations Research Techniques by the students of management cannot be overemphasized. Present book is an attempt to provide a means for such understanding by concise explanation of the concepts and numerous solved examples and illustrations.

This book: “*Elements of Operations Research*” has been designed as an introductory text to the world of Operations Research. Primarily, it has been written for B.Com and BBA students of Madras University and Guru Nanak Dev University, Amritsar, yet the book will be suitable for the similar undergraduate and postgraduate courses in Operations Research/Quantitative techniques of other universities.

The salient features of the book are:

- The book covers complete syllabus in Operations Research for BBA, B.Com and M.Com of almost all the Universities.
- There are detailed self-contained chapters on all the syllabus elements.
- The text uses simple language, lucid style, cogent presentation, and clarity of exposition.
- The text begins with chapter 1 highlighting the importance, modelling, methodology, applications and scope of Operations Research in business and industry.
- Chapter 2 concentrates on the formulation of practical problems (based on real life situations) in the format of linear programming problems. Chapter 3 and 4 deal with the development of solution methods of various linear programming problems.
- The special class of linear programming problems: Transportation and Assignment problems are discussed in Chapter 5 and 6 respectively.
- The concept of network in scheduling techniques using PERT/CPM methods are discussed in chapter 7.
- Decision-making rules under uncertainty, under risk and decision trees are discussed in chapter 8, while decision-making problems under conflict (Game Theory) are dealt with in chapter 9.
- Chapter 10 introduces the concept of queuing system, wherein elements of queuing process and Poisson Queuing Models are discussed.

- The concept of inventory in industry, its various types, and Economic Order Quantity (EOQ) are given in Chapter 11. EOQ models with and without price-breaks are discussed in detail.
- Chapter 12 is devoted to replacement decisions. This chapter also include problems related to recruitment and promotion.
- Chapter 13 on “Operations Scheduling” i.e., Sequencing has been dealt with at greater length owing to its increasing importance in the current business environment.

I express my gratitude to Dr. Priyanshu Gupta, Assistant Professor, Indian Institute of Management, Lucknow for his valuable suggestions during the writing of this book. It is earnestly hoped that the book will be found most useful and of greater utility to the students of B.Com, BBA and M.Com as well as to the business executives and consultants.

Suggestions for improvement shall be greatly received and duly acknowledged.

June, 2022

Dr. P K GUPTA

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SYLLABUS

UNIVERSITY OF MADRAS BBA Degree (General) Degree Course Syllabus *w.e.f.* 2020-2021

BBA-DSA04

Allied-IV: Operations Research

Year: II

Semester: IV

UNIT I : Introduction to Operations Research

Meaning and Scope - Characteristics - Models in Operations Research, LPP - Formulation – Graphical Method - Simplex Method - Big M Method - Applications in Business - Merits and Demerits.

UNIT II : Transportation Model

Basic Feasible Solution - Formulation, Solving a TP, Assignment Models - Formulation - Solution.

UNIT III : Network Analysis

Work break down analysis - Construction - Numbering of events - Time calculation - Critical path, slack, float - application.

UNIT IV : Queuing Models

Elements of queuing system - Characteristics of queuing model.

UNIT V : Decision Theory

Statement of Baye's Theorem Application, Probability - Decision Trees.

B.Com. (General) Degree Course Syllabus *w.e.f.* 2020-2021

BGE-CSA4A

Allied-IV(A): Elements of Operations Research

Common To B.Com(A&F), B.Com(MM), B.Com(CA) & B.Com(ISM)

Year: II

Semester: IV

Objectives:

- ✦ To facilitate this understanding of the Concept of Operations Research
- ✦ To help the Students to understand the Various Techniques of Solving Problems

Out Come:

- ✦ Understanding of the Concept of Operations Research and to help the Students to Understand the Various Techniques of Solving Problems

UNIT I : Introduction

Operations Research - Meaning-Definition - Origin and History- Characteristic Features - Need-Scope - Steps - Techniques - Application- Limitations

UNIT II : Linear Programming Problem LPP

Meaning - Requirements - Assumptions - Applications - Formulating LPP - Advantages - Limitations
Formulating LP Model (Simple Problems Only)

UNIT III : Methods of LPP

Obtaining Optimal Solution for Linear Programming Problem (LPP) - Graphical Method - Problems - Simplex Method for Type of LPP and for Slack Variable Case - Maximization Function - Minimization Function (Simple Problem Only)

UNIT IV : Transportation Problems

Meaning - (Initial Basic Feasible Solution) Assumptions - Degenerate Solution - North - West Corner Method - Least Cost Method - Vogels Approximation Method - Assignment Problems - Features - Transportation Problem Vs Assignment Problem - Hungarian Method (Simple Problems Only)

UNIT V : Game Theory

Meaning- Types of Games- Basic Assumptions- Finding Value of Game for Pure Strategy -Mixed Strategy -Indeterminate Matrix and Average Method -Graphical Method -Pure Strategy- Saddle Point Payoff Matrix Value of Game (Simple Problems Only)

GURU-NANAK DEV UNIVERSITY, AMRITSAR

Bachelor of Business Administration

Syllabus *w.e.f.* 2020-2021

BBA-406

Operations Research

Year: II

Semester: IV

Section A

Operations Research: Meaning, Significance and Scope, Introduction to Linear Programming, Formation of Linear Programming Problem, Graphical Method, Simple Method, Two Phase, Simplex Method, Duality in Linear Programming.

Section B

Definition of Dual Problem, General Rules of Converting Primal into its Dual, Transportation Problem, Assignment Problem.

Section C

Game Theory: Two persons zero - sum Games, Pure Strategies, Mixed Strategies, Dominance.

Inventory: Types, Nature and classification, Economic lot size models, Quantity Discounts.

Section D

CPM/PERT: Basic concepts of Network, Preparation of the Network diagram, Project Duration and Critical Path, Probability of Project Completion.

GURU-NANAK DEV UNIVERSITY, AMRITSAR

B.Com (Pass & Hons.) Syllabus *w.e.f.* 2020-2021

BCG-603

Operations Research

Year: II

Semester: IV

Basics of Operations Research: Development, Definition, Characteristics, Necessity, Scope, Limitations.

Section A

Linear Programming: Introduction, Applications, Formulation of Linear Programming Problem, General Linear Programming Problem, Graphical Method of Solution, Theory of Simplex Method, Big-M Method..

Section B

Transportation Model: Assumption, Formulation and Solution of Transportation Models, Trans-shipment Problems, Definition of Assignment Model, Hungarian Method for Solution of Assignment Problems, Travelling Salesman Problem.

Section C

Queuing Models: Application, Introduction, Elements, Operating Characteristics, Waiting Time and Idle time costs, Model I - Single Channel Poisson Arrival with Exponential Service Times (Infinite Population); Assumption & Limitation Poisson of .Queuing-Model.

Game Theory: Theory of Games, Characteristics of Games, Rules - Look for a pure strategy, Reduce Game by Dominance, Mixed Strategies, (2×2 games, $2 \times n$ games and $m \times 2$ games)

Section D

Network Analysis in Project Planning: Project, Project Planning Scheduling, CPM, PERT, Cost Analysis and Crashing the Network Activities.

UNIVERSITY OF KERALA

BBA

Syllabus *w.e.f.* 2020-2021

CO 244S

Management Optimization Techniques

Year: II

Semester: IV

Objectives:

1. To convey basic principles and application of optimization tools of resource utilization.
2. To provide an insight into optimal project implementation Techniques under deterministic and probabilistic conditions.

Model 1: Optimization Techniques: Introduction - Definition-Nature and Importance. Steps in Intelligent decision making. Modelling Techniques.

Model 2: Linear Programming: Introduction - Formulation of LPP - Maximize, Minimize, objective function constraints and non negativity conditions. Graphical solution - Optimal solution - Infeasible and unbounded solution degeneracy - Simplex method - slack surplus and artificial variable - Duality Primal and dual problem..

Model 3: Transportation and Assignment Problems: Transportation Problem - Initial basic solution using North West Corner rule - Lower Cost Entry Method and Vogel Approximation Method - Optimal solution MODI method. Degeneracy - Unbalanced and Maximization in Transportation Problems. Assignment Problems Hungarian Method Maximization. Unbalanced and Restricted Assignment Problems Travelling Salesman Problems.

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11.	Inventory Control	22	46	10	—	18
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	Total	210	389	142	69	196

About the Book

It has been widely acknowledged the importance of Operations Research/Quantitative Techniques in Managerial decision-making. The need for a thorough understanding of various Operations Research Techniques by the students of management cannot be overemphasized. Present book is an attempt to provide a means for such understanding by concise explanation of the concepts and numerous solved examples and illustrations.

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About the Author

Dr. PK Gupta has over three decades of experience in teaching Mathematics and Operations Research to undergraduate and postgraduate students. He obtained a Master's degree in Mathematics, and then another one in Operations Research from the University of Delhi. He obtained his doctorate in 1977 in the field of Queuing Theory.

He served as a faculty member in various capacities at the Department of Mathematics, JV Jain College, Saharanpur; where he taught a range of topics on Mathematics and Operations Research. These included Boolean Algebra, Linear Algebra, Difference Equations, Graph Theory and Techniques of Operations Research.

In 1979, he was invited by the Cochin University of Science & Technology, Kochi (Kerala) to teach the students of their newly started diploma course in Operations Research and Computer Applications. He also has been actively engaged in research in the discipline. He supervised multiple Ph.D. scholars in Optimization of Queues and also applied Optimization Techniques in paper industry. He has authored more than half a dozen books of Mathematics and Operations Research, in addition to multiple research publications.



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