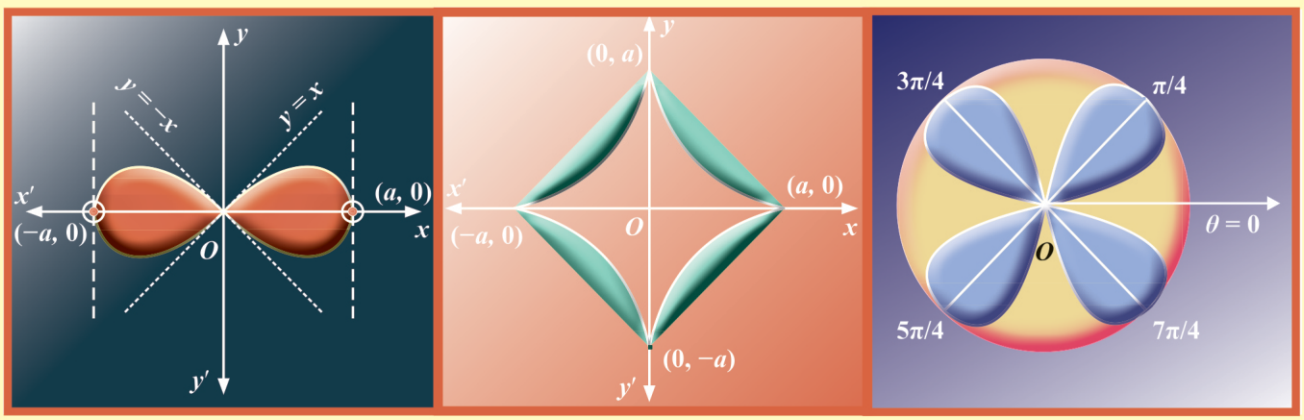
AS PER LATEST UGCF – 2022 PATTERN

A TEXTBOOK ON

FUNDAMENTALS OF CALCULUS

National Education Policy–2020



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A TEXTBOOK ON

FUNDAMENTALS OF CALCULUS

Dedicated to
Our Loving Parents

A TEXTBOOK ON

FUNDAMENTALS OF CALCULUS

(LATEST UGCF – 2022 PATTERN)

Based on National Education Policy 2020

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Educational Publishers
New Delhi

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Educational Publishers
23, Daryaganj, New Delhi-110 002
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011-40234454, 23247051 *(Office)*
E-mail : sultanchand74@yahoo.com; info@sultanchandandsons.com
Fax : 011-23266357; Website : www.sultanchandandsons.com

ISBN : 978-93-91820-01-5 (TC-1269)

Price **: ₹ 350.00**

First Edition: 2022

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Author's Acknowledgement: The writing of a Textbook always involves creation of a huge debt towards innumerable authors and publications. We owe our gratitude to all of them. We acknowledge our indebtedness in extensive footnotes throughout the book. If, for any reason, any acknowledgement has been left out we beg to be excused. We assure to carry out correction in the subsequent edition, as and when it is known.

Printer: Taj Printers, Noida

Preface

The authors take immense pleasure in presenting to the readers the book, entitled "Fundamentals of Calculus" created and compiled with an objective to provide the readers with a worthwhile experience of learning the basic concepts of Calculus.

This book has been designed in accordance with the Undergraduate Curriculum Framework-2022 followed by the Central Universities of India including University of Delhi under the National Education Policy (NEP)-2020. It is exclusively crafted to cater to the interests of students of Mathematics DSC-5 B.Sc (Hons.) Mathematics; GE-1(i) B.Sc./B.A. (Hons.)(Other than Mathematics); Discipline A-1 and GE-1(i) Bachelor in Multidisciplinary Courses. It is also useful for B.Tech. students of various Universities and for preparation of competitive examinations. The students of open and distance education courses will also find the book very beneficial.

A sincere and humble attempt has been made to provide the readers a complete and self-sufficient book written in a lucid and simplified manner giving comprehensive step-by-step explanations for better understanding of the subject.

There are Ten chapters in this book. In each, the concepts are vividly explained, supported by illustrations, followed by sufficient examples and exercises to provide the students an integrated view of the theory and its applications.

Most of the questions conform to the examination pattern followed in the University examinations and professional examinations.

We, gratefully, acknowledge the inspiration, encouragement and valuable suggestions received from the teachers who are teaching undergraduate and postgraduate courses of several Universities. In particular, we convey our thanks to Prof Rajiv Chopra (Principal, DCAC, University of Delhi), Prof Suman Sharma (Principal, Lady Shri Ram College for Women, University of Delhi), Prof Jatinder Bir Singh (Principal, Sri Guru Gobind Singh College of Commerce, University of Delhi), Prof Ayub Khan (Head of Department of Mathematics, Jamia Milia Islamia), Prof Ruchi Das (Head of Department of Mathematics, University of Delhi), Prof C K Jaggi (Head of Department of Operations Research, University of Delhi), Prof Rajiv Aggarwal, (Principal, Deshbandhu College, University of Delhi), Prof S.K Verma (Head of Department, SOL, University of Delhi), Prof Kul

Anand Sharma, Prof B.K Tyagi, Dr Satyendra Kumar, Dr (Mrs) Arun Bala Vaish. Gratitude is also due to our families for their unwavering support throughout this journey.

Our heartfelt acknowledgement is due to the entire staff of the publishers Sultan Chand and Sons for their efficiency, assistance, and wholehearted cooperation.

Genuine efforts have gone to proof read the book to avoid any errors. However, any misprint or inaccuracy left out inadvertently brought to our notice will be thankfully acknowledged.

New Delhi

Prof. (Dr.) Chaitanya Kumar
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Dr. Harinderjit Kaur Chawla

Syllabus

B.Sc. (Hons.) Mathematics (Sem II)
DSC-5: Calculus

Unit 1: Limits and Continuity

Limits of functions ($\varepsilon - \delta$ and sequential approach), Algebra of limits, Squeeze theorem, One-sided limits, Infinite limits and limits at infinity; Continuous functions and its properties on closed and bounded intervals; Uniform continuity.

Unit 2: Differentiability and Mean Value Theorems

Differentiability of a real-values functions, Algebra of differentiable functions, Chain rule, Relative extrema, Interior extremum theorem, Rolle's theorem, Mean-value theorem and its applications, Intermediate value theorem for derivatives.

Unit 3: Successive Differentiation, Taylor's Theorem and Tracing of Plane Curves

Higher order derivatives and calculation of the n^{th} derivative, Leibnitz's theorem; Taylor's theorem, Taylor's series expansions of e^x, $\sin x$, and $\cos x$; Indeterminate forms, L'Hôpital's rule; Concavity and inflexion points; Singular points, Asymptotes, Tracing graphs of rational functions and polar equations.

Bachelor of Multidisciplinary Courses of Study with 2 Core and 3 Core Courses (Sem I)
Discipline A-1: Topics in Calculus

Unit 1: Limits, Continuity and Differentiability

Limits of a functions, $\varepsilon - \delta$ definition of a limit, Infinite limits, Continuity and types of discontinuities; Differentiability of a function, Successive differentiation: Calculation of the nth derivatives, Leibnitz theorem; Partial differentiation, Euler's theorem on homogeneous functions.

Unit 2: Mean Value Theorems and its Applications

Rolle's theorem, Mean value theorems and applications of monotonic function and inequalities; Taylor's theorem, Taylor's series, Maclaurin's series expansions of e^x, $\sin x$, $\cos x$, $\log (1 + x)$ and $(1+x)^m$; Indeterminate forms.

Unit 3: Tracing of Curves and Reduction Formulae

Asymptotes (parallel to axes and oblique), Concavity and inflexion points, Singular points, Tangents at the origin and nature of singular points, Curve tracing (Cartesian and polar equations). Reduction formulae for $\int \sin^n x\,dx$, $\int \cos^n x\,dx$, and $\int \sin^m x \cos^n x\,dx$, and their applications.

B.Sc. (Hons)/B.A.(Hons) (other than Mathematics)(Sem I)
GE-1(i): Fundamentals of Calculus

Unit 1: Continuity and Differentiability of Functions

Limits of a functions, Types of discontinuities; Differentiability of functions, Successive differentiation: Calculation of the n^{th} derivatives, Leibnitz theorem; Partial differentiation, Euler's theorem on homogeneous functions.

Unit 2: Mean Value Theorems and its Applications

Rolle's theorem, Mean value theorems and applications of monotonic function and inequalities; Expansion of functions: Taylor's theorem, Taylor's series, Maclaurin's series expansions of e^x, $\sin x$, $\cos x$, $\log(1+x)$ and $(1+x)^m$; Indeterminate forms.

Unit 3: Tracing of Curves

Concavity and inflexion points, Asymptotes (parallel to axes and oblique), Relative extrema, Tracing graphs of polynomial functions, rational functions, and polar equations.

Contents

Snapshot of the Book

Chapter Number	*Chapter Title*	*Pages*	*Exercises*	*Theorems*	*Examples*
1	Limits, Continuity and Differentiability	44	28	10	46
2	Successive Differentiation	20	23	1	13
3	Partial Differentiation	24	32	—	18
4	Tangents and Normals	26	21	—	20
5	Asymptotes and Singular Points	14	18	—	12
6	Maxima and Minima	30	32	5	14
7	Curve Tracing	20	23	—	22
8	Mean Value Theorems	26	20	—	21
9	Indeterminate Forms	24	9	—	38
10	Reduction Formulae	32	38	—	27
	Total	**260**	**244**	**16**	**231**

Greek Alphabets

Alpha	A	,	α	Nu	N	,	ν
Beta	B	,	β	Xi	Ξ	,	ξ
Gamma	Γ	,	γ	Omicron	O	,	ο
Delta	Δ	,	δ	Pi	Π	,	π
Epsilon	E	,	ε	Rho	P	,	ρ
Zeta	Z	,	ζ	Sigma	Σ	,	σ
Eta	H	,	η	Tau	T	,	τ
Theta	Θ	,	θ	Upsilon	ϒ	,	υ
Iota	I	,	*ί*	Phi	Φ	,	ϕ
Kappa	K	,	κ	Chi	X	,	ξ
Lambda	Λ	,	λ	Psi	Ψ	,	ψ
Mu	M	,	μ	Omega	Ω	,	ω

Symbols

⇒	implies
⇔	is equivalent to
{ }	set
∈	is an element of
:	such that
⊂	is contained in (is a subset of)
⊃	contains (is a superset of)
$X \sim A$	complement of A with respect to X
∪	union
∩	intersection
Φ	the empty set
∃	there exists
∀	for all
$\mathbb{N}$	the set of all natural numbers
Z	the set of all integers
$\mathbb{Z}^+$	the set of all positive integers
Q	the set of all rational numbers
Q^+	the set of all positive rational numbers
$\mathbb{R}$	the set of all real numbers
$\mathbb{R}^+$	the set of all positive real numbers

Important Formulae

1. A function f is **continuous** at a point $x = c$, if

$$\lim_{x \to c-} f(x) = \lim_{x \to c+} f(x) = f(c).$$

2. (*i*) The **left hand derivative** a function f at a point $x = c$ is given by

$$Lf'(c) = \lim_{x \to c-} \frac{f(x) - f(c)}{x - c}, \text{ provided the limit exists.}$$

(*ii*) The **right hand derivative** of a function f at $x = c$ is given by

$$Rf'(c) = \lim_{x \to c+} \frac{f(x) - f(c)}{x - c}, \text{ provided the limit exists.}$$

(*iii*) The **derivative** of a function f at $x = c$, denoted by $f'(c)$, exists iff $Lf'(c) = Rf'(c)$ and in this case

$$f'(c) = Lf'(c) = Rf'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

3.
$$\frac{d^n}{dx^n}\left(\frac{1}{ax+b}\right) = \frac{(-1)^n\, n!\, a^n}{(ax+b)^{n+1}}$$

4.
$$\frac{d^n}{dx^n}\{\log(ax+b)\} = \frac{(-1)^{n-1}(n-1)!\, a^n}{(ax+b)^n}$$

5. (*i*)
$$\frac{d^n}{dx^n}\{\sin(ax \quad b)\} = a^n \sin\left(ax + b + \frac{n\pi}{2}\right)$$

(*ii*)
$$\frac{d^n}{dx^n}\{\cos(ax+b)\} = a^n \cos\left(ax + b + \frac{n\pi}{2}\right)$$

6. (*i*)
$$\frac{d^n}{dx^n}\left[e^{ax}\sin(bx+c)\right] = (a^2 + b^2)^{n/2}\, e^{ax} \sin\left(bx + c + n\tan^{-1}\frac{b}{a}\right)$$

(*ii*)
$$\frac{d^n}{dx^n}\left[e^{ax}\cos(bx+c)\right] = \left(a^2 + b^2\right)^{n/2} e^{ax} \cos\left(bx + c + n\tan^{-1}\frac{b}{a}\right)$$

7. (Leibnitz's Theorem)

$$(UV)_n = U_n V + {}^nC_1\, U_{n-1}\, V_1 + {}^nC_2\, U_{n-2}\, V_2 + \ldots + UV_n$$

where U and V are functions of x

8. (Euler's Theorem on Homogeneous Functions)
If $z = f(x, y)$ is a homogeneous function of x and y of degree n, then

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial x} = nz.$$

9. (*i*) The equation of the **tangent** to a curve at a point $P(x, y)$ on it is

$$Y - y = (dy/dx)(X - x).$$

(*ii*) The equation of the **normal** to a curve at a point $P(x, y)$ on it is

$$Y - y = -(dx/dy)(X - x).$$

10. The **angle 'ϕ'** between the radius vector and tangent is given by

$$\tan \phi = r\frac{d\theta}{dr}.$$

11. The **asymptotes parallel** *to the x-axis* (*y-axis*) are given by equating to zero the real linear factors in the coefficient of the highest power of x (y) in the equation of the given curve.

12. **Tangents at origin** are given by equating to zero the lowest degree terms in the equation of the given curve.

13. (*i*) The **double points** of a curve $f(x, y) = 0$ are given by solving the following equations:

$$f(x, y) = 0, \quad \frac{\partial f}{\partial x} = 0, \quad \frac{\partial f}{\partial y} = 0.$$

(*ii*) If $P(a, b)$ is a **double point** on the curve $f(x, y) = 0$, then it is a **node, cusp, conjugate point** according as

$$\left[\left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 - \frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 f}{\partial y^2}\right]_{P(a,b)} > 0, = 0, < 0.$$

14. Six Important Integrals

(*i*) $$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C$$

(*ii*) $$\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1}\frac{x}{a} + C$$

(*iii*) $$\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\frac{x}{a} + C$$

(*iv*) $$\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$$

(*v*) $$\int \sqrt{a^2 + x^2}\,dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\sinh^{-1}\frac{x}{a} + C$$

(*vi*) $$\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 + a^2} - \frac{a^2}{2}\cosh^{-1}\frac{x}{a} + C$$

15. $$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$$

16. $$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \frac{x-a}{x+a} + C$$

where C is a constant of integration

17. Methods of Integration of Some Irrational Functions

(*i*) $\int \frac{dx}{(Ax+B)\sqrt{ax+b}}$, Put $ax + b = t^2$.

(*ii*) $\int \frac{dx}{(Ax+B)\sqrt{ax^2+bx+c}}$, Put $Ax + B = \frac{1}{t}$

(*iii*) $\int \frac{dx}{(Ax^2+B)\sqrt{ax^2+b}}$, Put $\frac{ax^2+b}{Ax^2+B} = t^2$.

18. $$\int_0^{\pi/2} \sin^n x\, dx = \int_0^{\pi/2} \cos^n x\, dx = \begin{cases} \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{3}{2}, \\ \text{if } n \text{ an odd integer} \\ \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{1}{2} \cdot \frac{\pi}{2}, \\ \text{if } n \text{ an even integer} \end{cases}$$

19. $$\int_0^{\pi/2} \sin^m x \cos^n x\, dx = \frac{(m-1)(m-3)\ldots(n-1)(n-3)\ldots}{(m+n)(m+n-2)\ldots}$$

to be multiplied by $\frac{\pi}{2}$, when m and n are both positive even integers.

About the Book

This book has been designed in accordance with the Undergraduate Curriculum Framework-2022 followed by the Central Universities of India including University of Delhi under the National Education Policy (NEP)-2020. Keeping in mind the need to uphold students' interest in the subject, vivid explanation of concepts as well as explanatory illustrations followed by exercises have been included. The book is exclusively designed to help and guide the students of Mathematics DSC-5 B.Sc. (Hons.) Mathematics; GE-1(i) B.Sc./B.A. (Hons.) (Other than Mathematics); Discipline A-1 and GE-1(i) Bachelor in Multidisciplinary Courses. It is also useful for B.Tech. students of various Universities and for preparation of competitive examinations. The students of open and distance education courses will also find the book very beneficial.

Salient Features

- *An all-encompassing and self-sufficient textbook* for UGCF-2022 based on NEP-2020.
- Written in a *lucid and simple language.*
- Written with a view to present a *qualitative understanding* of the subject.
- *Comprehensive step-by-step explanation* for easier understanding of the subject.
- Many *solved examples and unsolved problems* have been drawn from recent examination papers of Universities.
- *Answers to all the problems in each exercise* have been provided immediately after the exercise for the convenience of the reader.
- Recent *Delhi University Question Papers with Solutions* have been included for ample practice.

About the Authors

Prof. (Dr.) Chaitanya Kumar has been teaching Mathematics in Delhi College of Arts and Commerce, University of Delhi for the last 40 years. He received his Doctorate degree in Mathematics from University of Delhi. He has published several research papers in the area of Boundary Value Problems by Integral Equation Techniques in reputed International Journals. He has been a Research Supervisor at Post Graduate and higher levels. He has authored important books on Mathematics; Essentials of Mathematics for Business Studies, Mathematics for M.C.A. Entrance Examination, Basic Research Methods and Statistics for Social Sciences, Theory of Analytic Geometry and Applied Algebra, Elements of Analysis, Ordinary and Partial Differential Equations for graduate and higher levels.

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ISBN 978-93-91820-01-5