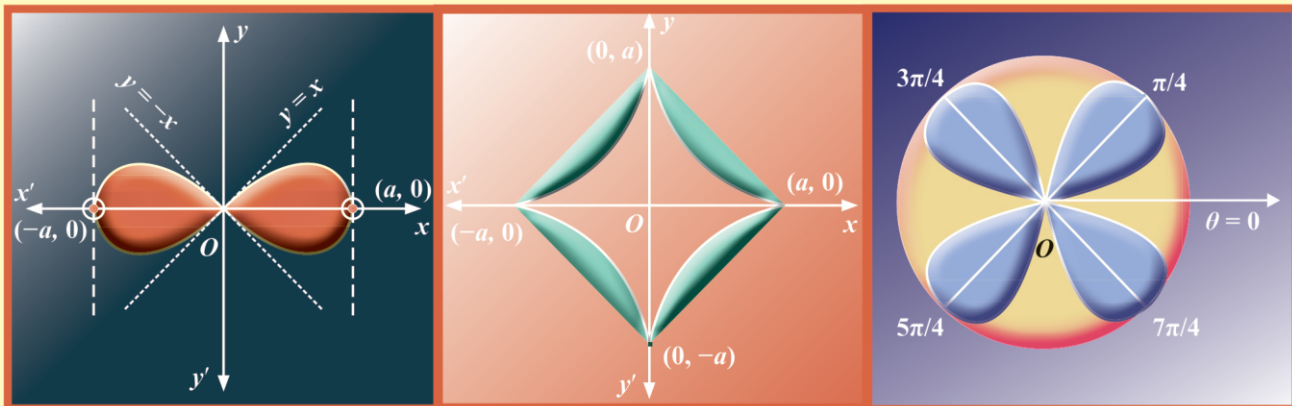




AS PER LATEST UGCF – 2022 PATTERN

A TEXTBOOK ON
**FUNDAMENTALS
OF CALCULUS**

National Education Policy–2020



Prof. Chaitanya Kumar

Dr. Bhavneet Kaur

Dr. Harinderjit Kaur Chawla

SULTAN CHAND & SONS

A TEXTBOOK ON
FUNDAMENTALS
OF CALCULUS

Dedicated to
Our Loving Parents

A TEXTBOOK ON
**FUNDAMENTALS
OF CALCULUS**

(LATEST UGCF – 2022 PATTERN)
Based on National Education Policy 2020

PROF. CHAITANYA KUMAR

Professor
Delhi College of Arts and Commerce
University of Delhi, Delhi

DR. BHAVNEET KAUR

Associate Professor
Lady Shri Ram College for Women
University of Delhi, Delhi

DR. HARINDERJIT KAUR CHAWLA

Associate Professor
Sri Guru Gobind Singh College of Commerce
University of Delhi, Delhi



SULTAN CHAND & SONS[®]

Educational Publishers
New Delhi

SULTAN CHAND & SONS®

Educational Publishers

23, Daryaganj, New Delhi-110 002

Phones : 011-23281876, 23266105, 23277843 (*Showroom & Shop*)

011-40234454, 23247051 (*Office*)

E-mail : sultanchand74@yahoo.com; info@sultanchandandsons.com

Fax : 011-23266357; Website : www.sultanchandandsons.com

ISBN : 978-93-91820-01-5 (TC-1269)

Price : ₹ 350.00

First Edition: 2022

EVERY GENUINE COPY OF THIS BOOK HAS A HOLOGRAM



In our endeavour to protect you against counterfeit/fake books, we have pasted a copper hologram over the cover of this book. The hologram displays the full visual image, unique 3D multi-level, multi-colour effects of our logo from different angles when tilted or properly illuminated under a single light source, such as 3D depth effect, kinetic effect, pearl effect, gradient effect, trailing effect, emboss effect, glitter effect, randomly sparkling tiny dots, micro text, laser numbering, etc.

'A fake hologram does not display all these effects.'

Always ask the bookseller to put his stamp on the first page of this book.

All Rights Reserved: No part of this book, including its style and presentation, may be reproduced, stored in a retrieval system, or transmitted in any form or by any means – electronic, mechanical, photocopying, recording or otherwise without the prior written consent of the Publishers. Exclusive publication, promotion and distribution rights reserved with the Publishers.

Warning: The doing of an unauthorised act in relation to a copyright work may result in both civil claim for damages and criminal prosecution.

Special Note: Photocopy or Xeroxing of educational books without the written permission of Publishers is illegal and against Copyright Act. Buying and selling of pirated books is a criminal offence. Publication of key to this is strictly prohibited.

General: While every effort has been made to present authentic information and avoid errors, the author and the publishers are not responsible for the consequences of any action taken on the basis of this book.

Limits of Liability/Disclaimer of Warranty: The publisher and the author make no representation or warranties with respect to the accuracy or completeness of the contents of this work and specifically disclaim all warranties, including without limitation warranties of fitness for a particular purpose. No warranty may be created or extended by sales or promotional materials. The advice and strategies contained herein may not be suitable for every situation. This work is sold with the understanding that the publisher is not engaged in rendering legal, accounting, or other professional services. If professional assistance is required, the services of a competent professional person should be sought. Neither the publisher nor the author shall be liable for damage arising herefrom.

Disclaimer: The publisher have taken all care to ensure highest standard of quality as regards typesetting, proofreading, accuracy of textual material, printing and binding. However, they accept no responsibility for any loss occasioned as a result of any misprint or mistake found in this publication.

Author's Acknowledgement: The writing of a Textbook always involves creation of a huge debt towards innumerable authors and publications. We owe our gratitude to all of them. We acknowledge our indebtedness in extensive footnotes throughout the book. If, for any reason, any acknowledgement has been left out we beg to be excused. We assure to carry out correction in the subsequent edition, as and when it is known.

Printer: Taj Printers, Noida



Preface

The authors take immense pleasure in presenting to the readers the book, entitled “Fundamentals of Calculus” created and compiled with an objective to provide the readers with a worthwhile experience of learning the basic concepts of Calculus.

This book has been designed in accordance with the Undergraduate Curriculum Framework-2022 followed by the Central Universities of India including University of Delhi under the National Education Policy (NEP)-2020. It is exclusively crafted to cater to the interests of students of Mathematics DSC-5 B.Sc (Hons.) Mathematics; GE-1(i) B.Sc./B.A. (Hons.)(Other than Mathematics); Discipline A-1 and GE-1(i) Bachelor in Multidisciplinary Courses. It is also useful for B.Tech. students of various Universities and for preparation of competitive examinations. The students of open and distance education courses will also find the book very beneficial.

A sincere and humble attempt has been made to provide the readers a complete and self-sufficient book written in a lucid and simplified manner giving comprehensive step-by-step explanations for better understanding of the subject.

There are Ten chapters in this book. In each, the concepts are vividly explained, supported by illustrations, followed by sufficient examples and exercises to provide the students an integrated view of the theory and its applications.

Most of the questions conform to the examination pattern followed in the University examinations and professional examinations.

We, gratefully, acknowledge the inspiration, encouragement and valuable suggestions received from the teachers who are teaching undergraduate and postgraduate courses of several Universities. In particular, we convey our thanks to Prof Rajiv Chopra (Principal, DCAC, University of Delhi), Prof Suman Sharma (Principal, Lady Shri Ram College for Women, University of Delhi), Prof Jatinder Bir Singh (Principal, Sri Guru Gobind Singh College of Commerce, University of Delhi), Prof Ayub Khan (Head of Department of Mathematics, Jamia Milia Islamia), Prof Ruchi Das (Head of Department of Mathematics, University of Delhi), Prof C K Jaggi (Head of Department of Operations Research, University of Delhi), Prof Rajiv Aggarwal, (Principal, Deshbandhu College, University of Delhi), Prof S.K Verma (Head of Department, SOL, University of Delhi), Prof Kul

Anand Sharma, Prof B.K Tyagi, Dr Satyendra Kumar, Dr (Mrs) Arun Bala Vaish. Gratitude is also due to our families for their unwavering support throughout this journey.

Our heartfelt acknowledgement is due to the entire staff of the publishers Sultan Chand and Sons for their efficiency, assistance, and wholehearted cooperation.

Genuine efforts have gone to proof read the book to avoid any errors. However, any misprint or inaccuracy left out inadvertently brought to our notice will be thankfully acknowledged.

New Delhi

Prof. (Dr.) Chaitanya Kumar
Dr. Bhavneet Kaur
Dr. Harinderjit Kaur Chawla

Syllabus



B.Sc. (Hons.) Mathematics (Sem II) **DSC-5: Calculus**

Unit 1: Limits and Continuity

Limits of functions ($\epsilon - \delta$ and sequential approach), Algebra of limits, Squeeze theorem, One-sided limits, Infinite limits and limits at infinity; Continuous functions and its properties on closed and bounded intervals; Uniform continuity.

Unit 2: Differentiability and Mean Value Theorems

Differentiability of a real-values functions, Algebra of differentiable functions, Chain rule, Relative extrema, Interior extremum theorem, Rolle's theorem, Mean-value theorem and its applications, Intermediate value theorem for derivatives.

Unit 3: Successive Differentiation, Taylor's Theorem and Tracing of Plane Curves

Higher order derivatives and calculation of the n^{th} derivative, Leibnitz's theorem; Taylor's theorem, Taylor's series expansions of e^x , $\sin x$, and $\cos x$; Indeterminate forms, L'Hôpital's rule; Concavity and inflexion points; Singular points, Asymptotes, Tracing graphs of rational functions and polar equations.

Bachelor of Multidisciplinary Courses of Study with 2 Core and 3 Core Courses (Sem I) **Discipline A-1: Topics in Calculus**

Unit 1: Limits, Continuity and Differentiability

Limits of a functions, $\epsilon - \delta$ definition of a limit, Infinite limits, Continuity and types of discontinuities; Differentiability of a function, Successive differentiation: Calculation of the n^{th} derivatives, Leibnitz theorem; Partial differentiation, Euler's theorem on homogeneous functions.

Unit 2: Mean Value Theorems and its Applications

Rolle's theorem, Mean value theorems and applications of monotonic function and inequalities; Taylor's theorem, Taylor's series, Maclaurin's series expansions of e^x , $\sin x$, $\cos x$, $\log(1+x)$ and $(1+x)^m$; Indeterminate forms.

Unit 3: Tracing of Curves and Reduction Formulae

Asymptotes (parallel to axes and oblique), Concavity and inflexion points, Singular points, Tangents at the origin and nature of singular points, Curve tracing (Cartesian and polar equations). Reduction formulae for $\int \sin^n x dx$, $\int \cos^n x dx$, and $\int \sin^m x \cos^n x dx$, and their applications.

B.Sc. (Hons)/B.A.(Hons) (other than Mathematics)(Sem I)
GE-1(i): Fundamentals of Calculus

Unit 1: Continuity and Differentiability of Functions

Limits of a functions, Types of discontinuities; Differentiability of functions, Successive differentiation: Calculation of the n^{th} derivatives, Leibnitz theorem; Partial differentiation, Euler's theorem on homogeneous functions.

Unit 2: Mean Value Theorems and its Applications

Rolle's theorem, Mean value theorems and applications of monotonic function and inequalities; Expansion of functions: Taylor's theorem, Taylor's series, Maclaurin's series expansions of e^x , $\sin x$, $\cos x$, $\log(1+x)$ and $(1+x)^m$; Indeterminate forms.

Unit 3: Tracing of Curves

Concavity and inflexion points, Asymptotes (parallel to axes and oblique), Relative extrema, Tracing graphs of polynomial functions, rational functions, and polar equations.



Contents

1. LIMITS, CONTINUITY AND DIFFERENTIABILITY	1–43
1.1. Introduction	1
1.2. Limit of a Function	1
1.2.1. Geometrical Significance of ε and δ	3
1.3. One Sided Limits	13
1.3.1. Right Hand Limit	13
1.3.2. Left Hand Limit	13
1.4. Infinite Limits and Limits at Infinity	16
1.4.1. Infinite Limits	16
1.4.2. Limits at Infinity	17
1.5. Continuity	23
1.5.1. Geometrical Interpretation of a Continuous Function	24
1.5.2. Discontinuous Functions	25
1.5.3. Types of Discontinuity	25
1.5.4. Algebra of Continuous Functions	32
1.6. Differentiability of a Function	36
1.6.1. Derivative of a Function at a Point	36
1.6.2. Relationship between Differentiability and Continuity	36
1.7. Uniform Continuity	40
1.7.1. Theorems on Uniform Continuity	42
2. SUCCESSIVE DIFFERENTIATION	45–64
2.1. Introduction	45
2.2. n^{th} Order Derivatives of Some Standard Functions	45
2.3. Leibnitz's Theorem	55
3. PARTIAL DIFFERENTIATION	65–88
3.1. Functions of Several Variables	65
3.2. Partial Derivatives	65

3.3	Homogeneous Functions	77
3.4	Euler's Theorem on Homogeneous Functions	78
3.4.1	Some Useful Deductions of Euler's Theorem	78
4.	TANGENTS AND NORMALS	89–114
4.1	Definition of Tangent and Normal	89
4.2	Equations of the Tangent and Normal	90
4.2.1	Cartesian Equations	90
4.2.2	Parametric Equations	91
4.3	Intercept on the Axes	92
4.4	Length of Tangent, Subtangent	92
4.5	Length of Normal, Sub-normal	93
4.6	Angle of Intersection of Two Curves	93
4.7	Tangents and Normals : Polar Coordinates	102
4.7.1	Angle between Radius Vector and Tangent	102
4.7.2	Angle of Intersection of Two Curves	103
4.8	Pedal Equations	107
4.8.1	Cartesian Equations	107
4.8.2	Length of the Perpendicular from the Pole to the Tangent	108
5.	ASYMPTOTES AND SINGULAR POINTS.....	115–128
5.1	Asymptotes	115
5.2	Determining the Oblique Asymptotes	115
5.3	Asymptotes Parallel to Coordinate Axis	117
5.4	Singular Point or Double Point	126
5.5	Tangent at the Origin	126
5.6	Position and Nature of Double Points or Singular Points	127
6.	MAXIMA AND MINIMA	129–148
6.1	Maximum and Minimum Values of a Function	129
6.2	Sufficient Condition for Extreme Values	132
6.3	Geometrical Problems of Maxima and Minima	137
6.4	Applied Max-Min Problems	137
6.5	Minimization of Average Cost	138
6.6	Maximization of Total Profit	139
6.7	Concavity, Convexity and Point of Inflexion	145
6.7.1	Point of Inflexion	145
6.7.2	Test for Concavity	145
7.	CURVE TRACING.....	149–178
7.1	Tracing of Cartesian Curves	149
7.1.1	Symmetry	149
7.1.2	The Origin and Tangents at Origin	150

7.1.3	Intersection with Co-ordinate Axes	150
7.1.4	Region(s) of Absence of the Curve	150
7.1.5	Maxima, Minima and Points of Inflexion	150
7.1.6	Asymptotes	150
7.1.7	Conversion to Polar Form	151
7.2	Curves in Parametric Form	159
7.3	Tracing of Polar Curves	166
8.	MEAN VALUE THEOREMS.....	179–203
8.1	Rolle’s Theorem	179
8.2	Lagrange’s Mean Value Theorem	183
8.2.1	Another Form of the Statement of Lagrange’s Mean Value Theorem	184
8.2.2	Problems on Inequalities	187
8.3	Cauchy’s Mean Value Theorem	189
8.4	Taylor’s Theorem	192
8.4.1	Taylor’s Theorem with Lagrange’s form of Remainder	192
8.4.2	Taylor’s Theorem with Cauchy’s form of Remainder	193
8.5	Maclaurin’s Theorem	194
8.6	Maclaurin’s Infinite Series	195
9.	INDETERMINATE FORMS	205–228
9.1	Indeterminate Forms	205
9.2	L’Hôpital’s Rule for $\left(\frac{0}{0}\right)$ Form	205
9.3	L’Hôpital’s Rule for $\left(\frac{\infty}{\infty}\right)$ Form	207
9.4	The Indeterminate Forms $0^0, 1^\infty$ & ∞^0	218
10.	REDUCTION FORMULAE	229–260
10.1	Reduction Formula for $\int \sin^n x \, dx$, n being a Positive Integer, $n \geq 2$	229
10.2	Reduction Formula for the Definite Integral $\int_0^{\pi/2} \sin^n x \, dx$, n being a Positive Integer, $n \geq 2$	231
10.3	Reduction Formula for $\int \cos^n x \, dx$, n being a Positive Integer, $n \geq 2$	232
10.4	Reduction Formula for the Definite Integral $\int_0^{\pi/2} \cos^n x \, dx$, n being a Positive Integer, $n \geq 2$	234
10.5	Reduction Formula for $\int \tan^n x \, dx$, n being a Positive Integer, $n \geq 2$	240
10.6	Reduction Formula for $\int \cot^n x \, dx$, n being a Positive Integer, $n \geq 2$	241
10.7	Reduction Formula for $\int \sec^n x \, dx$, n being a Positive Integer, $n \geq 2$	243

10.8	Reduction Formula for $\int \operatorname{cosec}^n x \, dx$, n being a Positive Integer, $b \geq 2$	243
10.9	Reduction Formula for $\int \sin^p x \cos^q x \, dx$ where p and q are Positive Integers	246
10.10	Reduction Formula for $\int \sin^p x \cos^q x \, dx$ where either p, q or both are Negative Integers	248
10.11	Integration of $\sin^p x \cos^q x$ where ' $p + q$ ' is a Negative Even Integer	251
10.12	Reduction Formula for the definite integral $\int_0^{\pi/2} \sin^p x \cos^q x \, dx$ where p and q are Positive Integers	255
UNIVERSITY QUESTION PAPERS WITH ANSWERS.....		261–318
GE-I : Nov 2017.....		261
GE-I : Nov 2018.....		275
GE-I : Nov 2019.....		291
GE-I : March 2021		299
GE-I : March 2022		308

Snapshot of the Book					
<i>Chapter Number</i>	<i>Chapter Title</i>	<i>Pages</i>	<i>Exercises</i>	<i>Theorems</i>	<i>Examples</i>
1	Limits, Continuity and Differentiability	44	28	10	46
2	Successive Differentiation	20	23	1	13
3	Partial Differentiation	24	32	—	18
4	Tangents and Normals	26	21	—	20
5	Asymptotes and Singular Points	14	18	—	12
6	Maxima and Minima	30	32	5	14
7	Curve Tracing	20	23	—	22
8	Mean Value Theorems	26	20	—	21
9	Indeterminate Forms	24	9	—	38
10	Reduction Formulae	32	38	—	27
	Total	260	244	16	231

Greek Alphabets

Alpha	A	,	α	Nu	N	,	ν
Beta	B	,	β	Xi	Ξ	,	ξ
Gamma	Γ	,	γ	Omicron	O	,	\omicron
Delta	Δ	,	δ	Pi	Π	,	π
Epsilon	E	,	ϵ	Rho	P	,	ρ
Zeta	Z	,	ζ	Sigma	Σ	,	σ
Eta	H	,	η	Tau	T	,	τ
Theta	Θ	,	θ	Upsilon	Υ	,	υ
Iota	I	,	ι	Phi	Φ	,	ϕ
Kappa	K	,	κ	Chi	X	,	χ
Lambda	Λ	,	λ	Psi	Ψ	,	ψ
Mu	M	,	μ	Omega	Ω	,	ω

Symbols

\Rightarrow	implies
\Leftrightarrow	is equivalent to
$\{ \}$	set
\in	is an element of
:	such that
\subset	is contained in (is a subset of)
\supset	contains (is a superset of)
$X \sim A$	complement of A with respect to X
\cup	union
\cap	intersection
Φ	the empty set
\exists	there exists
\forall	for all
\mathbb{N}	the set of all natural numbers
\mathbb{Z}	the set of all integers
\mathbb{Z}^+	the set of all positive integers
\mathbb{Q}	the set of all rational numbers
\mathbb{Q}^+	the set of all positive rational numbers
\mathbb{R}	the set of all real numbers
\mathbb{R}^+	the set of all positive real numbers

Important Formulae

1. A function f is **continuous** at a point $x = c$, if

$$\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) = f(c).$$

2. (i) The **left hand derivative** a function f at a point $x = c$ is given by

$$Lf'(c) = \lim_{x \rightarrow c^-} \frac{f(x) - f(c)}{x - c}, \text{ provided the limit exists.}$$

- (ii) The **right hand derivative** of a function f at $x = c$ is given by

$$Rf'(c) = \lim_{x \rightarrow c^+} \frac{f(x) - f(c)}{x - c}, \text{ provided the limit exists.}$$

- (iii) The **derivative** of a function f at $x = c$, denoted by $f'(c)$, exists iff $Lf'(c) = Rf'(c)$ and in this case

$$f'(c) = Lf'(c) = Rf'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$$

$$3. \quad \frac{d^n}{dx^n} \left(\frac{1}{ax + b} \right) = \frac{(-1)^n n! a^n}{(ax + b)^{n+1}}$$

$$4. \quad \frac{d^n}{dx^n} \{ \log(ax + b) \} = \frac{(-1)^{n-1} (n-1)! a^n}{(ax + b)^n}$$

$$5. \quad (i) \quad \frac{d^n}{dx^n} \{ \sin(ax + b) \} = a^n \sin \left(ax + b + \frac{n\pi}{2} \right)$$

$$(ii) \quad \frac{d^n}{dx^n} \{ \cos(ax + b) \} = a^n \cos \left(ax + b + \frac{n\pi}{2} \right)$$

$$6. \quad (i) \quad \frac{d^n}{dx^n} [e^{ax} \sin(bx + c)] = (a^2 + b^2)^{n/2} e^{ax} \sin \left(bx + c + n \tan^{-1} \frac{b}{a} \right)$$

$$(ii) \quad \frac{d^n}{dx^n} [e^{ax} \cos(bx + c)] = (a^2 + b^2)^{n/2} e^{ax} \cos \left(bx + c + n \tan^{-1} \frac{b}{a} \right)$$

7. **(Leibnitz's Theorem)**

$$(UV)_n = U_n V + {}^n C_1 U_{n-1} V_1 + {}^n C_2 U_{n-2} V_2 + \dots + UV_n$$

where U and V are functions of x

8. **(Euler's Theorem on Homogeneous Functions)**

If $z = f(x, y)$ is a homogeneous function of x and y of degree n , then

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = nz.$$

9. (i) The equation of the **tangent** to a curve at a point $P(x, y)$ on it is

$$Y - y = (dy/dx)(X - x).$$

- (ii) The equation of the **normal** to a curve at a point $P(x, y)$ on it is

$$Y - y = -(dx/dy)(X - x).$$

10. The **angle** ' ϕ ' between the radius vector and tangent is given by

$$\tan \phi = r \frac{d\theta}{dr}.$$

11. The **asymptotes parallel** to the x -axis (y -axis) are given by equating to zero the real linear factors in the coefficient of the highest power of x (y) in the equation of the given curve.

12. **Tangents at origin** are given by equating to zero the lowest degree terms in the equation of the given curve.

13. (i) The **double points** of a curve $f(x, y) = 0$ are given by solving the following equations:

$$f(x, y) = 0, \quad \frac{\partial f}{\partial x} = 0, \quad \frac{\partial f}{\partial y} = 0.$$

- (ii) If $P(a, b)$ is a **double point** on the curve $f(x, y) = 0$, then it is a **node, cusp, conjugate point** according as

$$\left[\left(\frac{\partial^2 f}{\partial x \partial y} \right)^2 - \frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 f}{\partial y^2} \right]_{P(a,b)} > 0, = 0, < 0.$$

14. Six Important Integrals

(i)
$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$$

(ii)
$$\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1} \frac{x}{a} + C$$

(iii)
$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1} \frac{x}{a} + C$$

(iv)
$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

(v)
$$\int \sqrt{a^2 + x^2} dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \sinh^{-1} \frac{x}{a} + C$$

(vi)
$$\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \cosh^{-1} \frac{x}{a} + C$$

15.
$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$16. \quad \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \frac{x-a}{x+a} + C$$

where C is a constant of integration

17. Methods of Integration of Some Irrational Functions

$$(i) \quad \int \frac{dx}{(Ax+B)\sqrt{ax+b}}, \quad \text{Put } ax+b = t^2.$$

$$(ii) \quad \int \frac{dx}{(Ax+B)\sqrt{ax^2+bx+c}}, \quad \text{Put } Ax+B = \frac{1}{t}$$

$$(iii) \quad \int \frac{dx}{(Ax^2+B)\sqrt{ax^2+b}}, \quad \text{Put } \frac{ax^2+b}{Ax^2+B} = t^2.$$

$$18. \quad \int_0^{\pi/2} \sin^n x \, dx = \int_0^{\pi/2} \cos^n x \, dx = \begin{cases} \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \dots \frac{3}{2}, \\ \text{if } n \text{ an odd integer} \\ \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \dots \frac{1}{2} \cdot \frac{\pi}{2}, \\ \text{if } n \text{ an even integer} \end{cases}$$

$$19. \quad \int_0^{\pi/2} \sin^m x \cos^n x \, dx = \frac{(m-1)(m-3)\dots(n-1)(n-3)\dots}{(m+n)(m+n-2)\dots}$$

to be multiplied by $\frac{\pi}{2}$, when m and n are both positive even integers.

About the Book

This book has been designed in accordance with the Undergraduate Curriculum Framework-2022 followed by the Central Universities of India including University of Delhi under the National Education Policy (NEP)-2020. Keeping in mind the need to uphold students' interest in the subject, vivid explanation of concepts as well as explanatory illustrations followed by exercises have been included. The book is exclusively designed to help and guide the students of Mathematics DSC-5 B.Sc. (Hons.) Mathematics; GE-1(i) B.Sc./B.A. (Hons.) (Other than Mathematics); Discipline A-1 and GE-1(i) Bachelor in Multidisciplinary Courses. It is also useful for B.Tech. students of various Universities and for preparation of competitive examinations. The students of open and distance education courses will also find the book very beneficial.

Salient Features

- *An all-encompassing and self-sufficient textbook* for UGCF-2022 based on NEP-2020.
- *Written in a lucid and simple language.*
- *Written with a view to present a qualitative understanding* of the subject.
- *Comprehensive step-by-step explanation* for easier understanding of the subject.
- *Many solved examples and unsolved problems* have been drawn from recent examination papers of Universities.
- *Answers to all the problems in each exercise* have been provided immediately after the exercise for the convenience of the reader.
- *Recent Delhi University Question Papers with Solutions* have been included for ample practice.

About the Authors

Prof. (Dr.) Chaitanya Kumar has been teaching Mathematics in Delhi College of Arts and Commerce, University of Delhi for the last 40 years. He received his Doctorate degree in Mathematics from University of Delhi. He has published several research papers in the area of Boundary Value Problems by Integral Equation Techniques in reputed International Journals. He has been a Research Supervisor at Post Graduate and higher levels. He has authored important books on Mathematics; Essentials of Mathematics for Business Studies, Mathematics for M.C.A. Entrance Examination, Basic Research Methods and Statistics for Social Sciences, Theory of Analytic Geometry and Applied Algebra, Elements of Analysis, Ordinary and Partial Differential Equations for graduate and higher levels.



Dr. Bhavneet Kaur is an Associate Professor, Department of Mathematics, Lady Shri Ram College for Women. She has an experience of 19 years of teaching undergraduate students of University of Delhi. She received her Doctorate degree in Mathematics from University of Delhi in 2016 and is actively involved in research in the field of Celestial Mechanics and Space Dynamics. Her work is published in National and International Journals of repute. She is also a research guide and is fully committed to the welfare of the student fraternity.

Dr. Harinderjit Kaur Chawla is an Associate Professor, Department of Mathematics, Sri Guru Gobind Singh College of Commerce, University of Delhi. She has been teaching Mathematics for the last 28 years. Dr. Chawla received her Doctorate degree in Mathematics from the University of Rajasthan in 1994. She has attended several National and International Conferences and is a life member of 'The Indian Science Congress Association'.



Sultan Chand & Sons

Publishers of Standard Educational Textbooks

23 Daryaganj, New Delhi-110002

Phones (O) : 011-23247051, 40234454

(S) : 011-23281876, 23266105, 41625022

Email : sultanchand74@yahoo.com

info@sultanchandandsons.com



TC-1269

ISBN 978-93-91820-01-5

