According to
NATIONAL EDUCATION POLICY 2020

Elements of Discrete Mathematics

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[As per National Education Policy (NEP-2020)]

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Author's Acknowledgement: The writing of a Textbook always involves creation of a huge debt towards innumerable author's and publications. We owe our gratitude to all of them. We acknowledge our indebtedness in extensive footnotes throughout the book. If, for any reason, any acknowledgement has been left out we beg to be excused. We assure to carry out correction in the subsequent edition, as and when it is known.

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Preface

The primary objective of this book is to provide the basic concepts of Discrete Mathematics as per the requirement of the students appearing for Undergraduate examination of B.Sc. (Hons) Mathematics, Bachelor in Multidisciplinary Courses of Study with three Core Disciplines or two Core Disciplines and Generic Elective of all disciplines other than Mathematics under 'New Education Policy (NEP-2020)' and B.Sc. (Hons) Mathematics and BA (P) under 'Choice-based Credit System (CBCS)' pattern followed by Central Universities of India, including University of Delhi.

As compared to other books available on the subject, we believe that this book is more elementary and written in such a way that the reader can understand and practice with a variety of examples.

There are seven chapters in this book. Chapter 1 and 2 give some basic definitions and results related to Number Systems and Set Theory respectively. The next two chapters are devoted to Lattices and Types of Lattices. Chapter 5 is about Logical Mathematics. Chapter 6 deals with results pertaining to Boolean Algebras. In the last chapter, Switching Circuits are explained, which are one of the major applications of Boolean Algebra. In the end of each chapter, large number of self practice problems have been added which will help the readers to understand the subject in a better way.

We must concede that this book would never have been written without the support and encouragement of our family members. Many thanks to them.

We have tried our best to keep the book free from errors. Still, if you find any potential error or wish to seek any clarification, please write to us. Your suggestions and comments for the improvement of this book will be thankfully received and duly incorporated in the subsequent edition.

Mamta Chaudhary
Vani Sharma
Pooja Yadav

Acknowledgment

I express my sincere thanks to Dr. Vani and Dr. Pooja for accompanying me in writing this book. I extend my thanks to Chaitanya Sir for encouraging me to write this book. In the end I thanks my husband from the bottom of my heart for his complete support in enabling me to complete this book.

Mamta Chaudhary

I am highly thankful to my co-authors Dr. Mamta Chaudhary and Dr. Pooja Yadav for their motivation and fruitful discussion which helped in improving the content of the book. I also gratefully acknowledge the encouragement and constant support of my family members, which helped me in completing the book.

Vani Sharma

Firstly, I would like to thank Dr. Mamta for encouraging and motivating me to contribute in this book. My sincere gratitude to all the authors who have written the books on *Elements of Discrete Mathematics* specially *Kenneth H. Rosen* and *Goodaire & Parmenter.* Their visions inspire and guide me to write this book. Good wishes and blessings are always behind the accomplishment of everything. So, I would like to thank all my family members and friends for all their support and wishes.

Pooja Yadav

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Syllabus

University of Delhi

Undergraduate Curriculum Framework (UGCF) – 2022

Based On

National Education Policy 2020

Mathematics Bachelor with 2 Core Disciplines DSC-1 (Sem-I)

B.Sc. Maths (H) for DSC-9 (Sem-III)

Bachelor in Multidisciplinary Courses with 3 Core Courses DSE-2 (*i*) (Sem-IV)

Unit 1: Sets, Relations and Functions

Sets, Propositions and logical operations, Conditional statements, Mathematical induction, Relations and equivalence relation, Equivalence classes, Partial order relation, Partially ordered set, Hasse diagrams, Chain, Maximal and minimal elements, Least and greatest elements, Least upper bound, Greatest lower bound, Zorn's Lemma, Functions and bijective functions, Functions between POSETS, Order isomorphism.

Unit 2: Lattices

Lattice as a POSET, Lattice as an Algebra and their equivalence, Bounded lattice, Sublattice, Interval in a lattice, Products and homomorphism of lattices, Isomorphism of lattices; Distributive, Complemented, Partition and Pentagonal lattices.

Unit 3: Boolean Algebra and Switching Circuits

Boolean Algebra, DeMorgan's Laws, Boolean expressions, Truth tables, Logic diagrams, Boolean functions, Disjunctive normal forms (as join of meets), Minimal forms of Boolean polynomials, Quine Mc-Cluskey method, Karnaugh maps, Switching circuits, Applications of switching circuits.

CBCS BSc (Hons.)

DSE-2 (*ii*): Discrete Mathematics

Unit 1: Ordered Sets

Definitions, Examples and basic properties of ordered sets, Order isomorphism, Hasse diagrams, Dual of an ordered set, Duality principle, Maximal and minimal elements, Building new ordered sets, Maps between ordered sets.

Unit 2: Lattices

Lattices as ordered sets, Lattices as algebraic structures, Sublattices, Products and homomorphisms; Definitions, examples and properties of modular and distributive lattices, The $M_3 - N_5$ Theorem with applications, Complemented lattice, Relatively complemented lattice, Sectionally complemented lattice.

Unit 3: Boolean Algebras and Switching Circuits

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Unit 4: Graph Theory

Introduction to graphs, Konigsberg Bridge problem, Instant insanity game; Definition, examples and basic properties of graphs, Subgraphs, Pseudographs, Complete graphs, Bipartite graphs, Isomorphism of graphs, Paths and circuits, Eulerian circuits, Hamiltonian cycles, Adjacency matrix, Weighted graph, Travelling salesman problem, Shortest path, Dijkstra's algorithm.

CBCS B.A. (Programme)

DSE-1 (*ii*): Discrete Mathematics

Unit 1: Partial Ordering

Definition, Examples and properties of posets, Maps between posets, Algebraic lattice, Lattice as a poset, Duality principle, Sublattice, Hasse diagrams; Products and homomorphisms of lattices, Distributive lattice, Complemented lattice.

Unit 2: Boolean Algebra and Switching Circuits

Boolean algebra, Boolean polynomial, CN form, DN form; Simplification of Boolean polynomials, Karnaugh diagram; Switching circuits and its applications, Finding CN form and DN form.

Unit 3: Graph Theory

Graphs, Subgraph, Complete graph, Bipartite graph, Degree sequence, Euler's theorem for sum of degrees of all vertices, Eulerian circuit, Seven bridge problem, Hamiltonian cycle, Adjacency matrix, Dijkstra's shortest path algorithm (improved version), Digraphs; Definitions and examples of tree and spanning tree, Kruskal's algorithm to find the minimum spanning tree; Planar graphs, Coloring of a graph and chromatic number.

University of Madras

B.Sc Degree Course in Mathematics

Core XII-Discrete Mathematics (Sem-V)

Unit 1

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Unit 2

Boolean algebra & Applications: Boolean algebra, Two element Boolean algebra, Disjunctive normal form, Conjunctive normal form

Unit 3

Application, Simplication of circuits, Designing of switching circuits, Logical gates and Combinatorial circuits.

Unit 4

Recurrence relations and Generating functions: Sequence and recurrence relation, Solving recurrence relations by iteration method, Modeling of counting problems by recurrence relations, Linear (difference equations) recurrence relations with constant coefficients, Generating functions, Sum and product of two generating functions, Useful generating functions, Combinatorial problems.

Unit 5

Proportional logic and Predicate logic: Proportional logic, Adequate

system of connectivies, Translation of sentences in a Natural language into Statement formula, Logical validity of arguments, Predicate Logic, Negation of a statement obtained by qualification of a predicate, Logical operations on predicates or quantified predicates, Symbolization of sentences by using predicates, Quantifiers and connectives, Logical validity of arguments.

Chaudhary Charan Singh University, Meerut

Degree in Mathematics
Semester V

Unit 1

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Unit 2

Relation – Definition, types of relation, domain and range of a relation, pictorial representation of relation, properties of relation, partial ordering relation. Representation of POSETS using Hasse diagram, Chains, Maximal and Minimal point. Glb, lub, Lattices and Algebraic system, Basic properties, Sublattices.

Unit 3

Boolean Algebra – Basic definitions, Sum of products and products of sums, Boolean Functions, Disjunctive normal form, Complete Disjunctive normal form, conjugate normal form, Logic circuits, Logic networks, Design of circuits from given properties, Logic gates, and Karnaugh maps.

Unit 4

Combinatorics – Inclusion- exclusion, recurrence relations (nth order recurrence relation with constant coefficients, Homogeneous recurrence relations, Inhomogeneous recurrence relations), generating function (closed form expression, properties of G.F., solution of recurrence relations using G.F. solution of combinatorial problem using G.F.

Glossary of Symbols

$\in$	:	Belongs to
$\notin$	:	Does not belong to
$\forall$	:	For all
$\exists$	:	There exists
$\Rightarrow$	:	Implies
$\Leftrightarrow$	:	Implies and is implied by
iff	:	If and only if
$\wedge$	:	Meet (and)
$\vee$	:	Join (or)
o(A)	:	Order of set A
$a \mid b$	:	a divides b
φ	:	Empty set
$\cong$	:	Is isomorphic to
$P(X)$	:	Set of all subsets of X
N	:	Set of natural numbers
Z	:	Set of integers
■	:	End of proof
$\nexists$	:	Does not exist
$\sim$	:	Equivalent to
$\nsim$	:	Not equivalent to
$\sum$	:	Sum
$\prod$	:	Product
$\rightarrow$	:	Implication

$\leftrightarrow$	:	Bi-implication
$'$	:	Negation or Complementation
$\subseteq$	:	Is a subset of
$\subset$	:	Is a proper subset of
$\supseteq$	:	Is a superset of
$\cup$	:	Union
$\cap$	:	Intersection
$\setminus$	:	Set difference
A^c	:	The complement of A
$\lvert A \rvert$	:	The Cardinality of A (if A is finite, then it is number of elements in A)
$\lvert a \rvert$	:	Absolute value of a

About the Book

This book "Elements of Discrete Mathematics" is primarily written for the Undergraduate students of different courses under CBCS and NEP followed in various universities in India including University of Delhi. The book is written for Multidisciplinary courses including Honors and Professional Courses as it covers both theoretical and applied approaches in simple and easy to understand language. Every topic is explained with relevant examples to grasp the idea behind the theory. Applications based questions are discussed to relate the importance of the subject.

Salient Features

- All the prerequisites are available in the book.
- Concepts are well explained and illustrations are done with examples for the better understanding of the basics.
- Exercises include conceptual, logical as well as some brainstorming questions for the readers.
- Syllabi under CBCS and NEP of relevant subject for different courses are included in the book with question papers.
- More than a 100 Miscellaneous Questions and 5 year Question Papers have been added to give ample practice to the students.

About the Authors

Dr. Mamta Chaudhary is an Associate Professor in the Department of Mathematics, Satyawati College. She brings 25 plus years of rich teaching experience in different courses in Delhi University and has taught Mathematical Analysis, Differential Equations, Algebra, Discrete Mathematics, Graph Theory and Metric Spaces at undergraduate level. She completed her doctorate in 2018 in Mathematical Programming from the University of Delhi and is still pursuing research work.

Dr. Vani Sharma is an Associate Professor in the Department of Mathematics, Satyawati College, University of Delhi. She has an experience of 23 years of teaching Undergraduate and Postgraduate students of University of Delhi. She is an alumna of Ramjas College and has been college topper for five years continuously. Dr. Vani has published several research papers in National and International journals. She is a lifetime member of National Research Group, Operational Research Society of India and International Research Group (Working Group of Generalized Convexity).

Dr. Pooja Yadav is working as an Assistant Professor in the Department of Mathematics, Kamala Nehru College, University of Delhi since 2010. In her 12 years experience, she has taught almost all the papers of Discrete Mathematics. She did her Ph.D in 2010 in Algebra from Indian Institute of Delhi (IITD). She was awarded scholarship by national Board of Higher mathematics (NBHM). Currently, she is supervising the Ph.D. students as well.

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