According to
NATIONAL EDUCATION POLICY 2020

# INTRODUCTION TO GRAPH THEORY

Mamta Chaudhary
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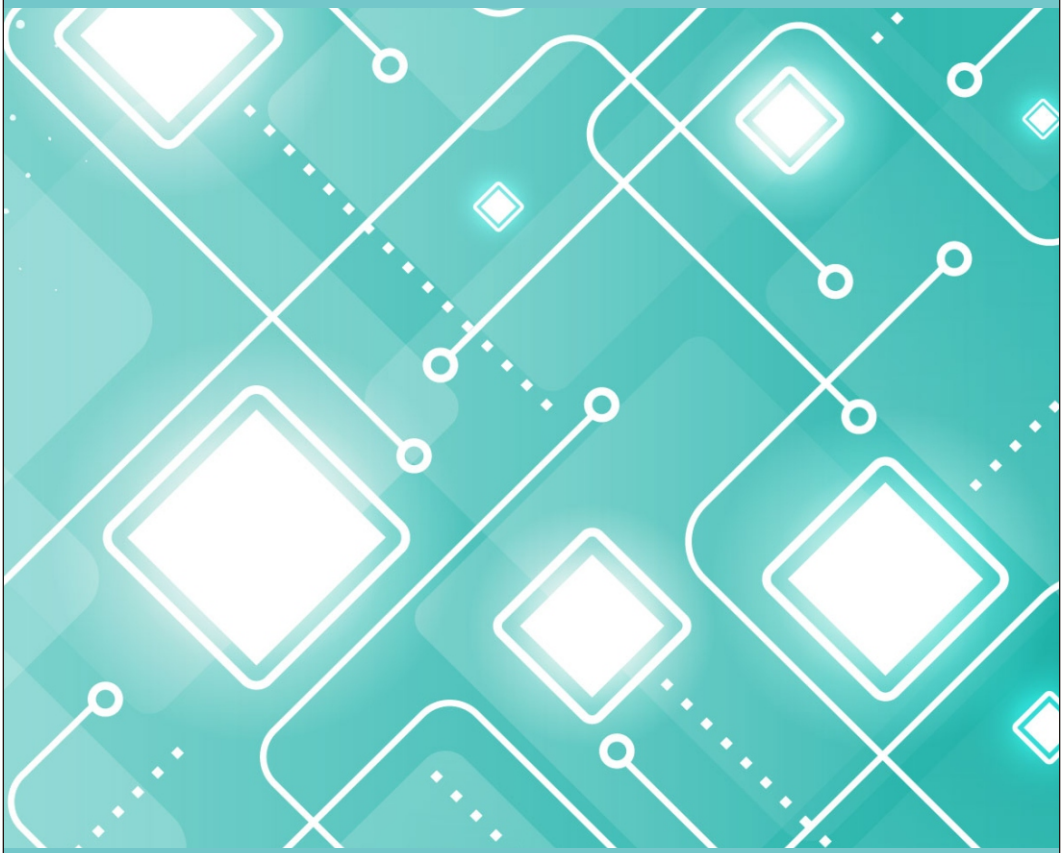
Pooja Yadav



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# Introduction to Graph Theory

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As per National Education Policy (NEP-2020)

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# Preface

This book fairly introduces the concepts of graph theory. The major goal of this book is to help the reader to understand the concepts of graphs. To achieve this goal, a clear statement of definitions, models, theorems and algorithms together with large number of solved examples are included. The examples mostly taken from various books and examination papers, serve to illustrate and amplify the theory. To review completely the subject matter of each chapter, a large number of supplementary exercises, are included in each chapter. At the end of the chapters, some important questions with solutions are also added. We have also included some material that finds applications of some graphs in other branches of mathematics.

With one exception, this book requires virtually no background. The exception is the adjacency matrix of a graph, where we assume a little linear algebra. Also this book completely meets the requirements of Mathematics Honours, Multidisciplinary Courses with two Core Disciplines or three Core Disciplines and Generic Elective of all disciplines other than Mathematics under 'New Education Policy (NEP-2020)' and under 'Choice-Based System (CBCS)' pattern followed by Central Universities of India, including University of Delhi.

The book consists of eight chapters. The chapter 1 provides the basic concepts of graph theory and presents many different graph models. Chapter 2 is divided into two parts. In the first part we will study how adjacency matrix and incidence matrix of graph represents the graph. Determining whether two graphs are isomorphic is an important problem of graph theory that we will study in the second part of this chapter. Chapter 3 explains relationship between different

types of walk for a pseudograph namely path, trail, cycle and circuit. Further in this chapter Eulerian circuits and Hamiltonian cycles are defined and explained with the help of various examples. Chapter 4 deals with various algorithms to find the shortest path in a weighted graph. Trees can be treated as data structures as well. Depth of the concepts of trees with applications is discussed in chapter 5. In chapter 6 planar representation of graphs and coloring of graphs are explained. Chapter 7 discusses Networks, Flows and Cuts. Chapter 8 focusses on various applications of graphs in different areas. It will give vision to readers for further study of graphs.

Finally, we put on record that without the encouragement and support of our family members, this book would have remained only a dream. The authors will appreciate recieving the intimation of misprints/misconcepts and shall duly acknowledge such intimations.

***Mamta Chaudhary***
***Vani Sharma***
***Pooja Yadav***

# Acknowledgment

I express my sincere thanks to Dr. Vani and Dr. Pooja for accompanying me in writing this book. I extend my thanks to Pratap Vaish Sir for constantly guiding us while writing this book. In the end I thank my husband from the bottom of my heart for his complete support in enabling me to complete this book.

*Mamta Chaudhary*

I am highly thankful to my co-authors Dr. Mamta Chaudhary and Dr. Pooja Yadav for their motivation and fruitful discussion which helped in improving the content of the book. I also gratefully acknowledge the encouragement and constant support of my family members, which helped me in completing the book.

*Vani Sharma*

Firstly, I would like to thank Dr. Mamta and Dr. Vani for their continuous support and patience. My sincere gratitude to all the authors who have written the books on *Graph Theory* specially *Kenneth H. Rosen* and *Goodaire & Parmenter.* Their visions inspire and guide me to write this book. I would like to thank all my students whose faith and zeal towards the subject motivated me to do this. Good wishes and blessings are always behind the accomplishment of everything. So, I would like to thank all my family members and friends for all their support and wishes.

*Pooja Yadav*

# Snapshot of the Book

| S. No. | Chapters | Pages | Definitions | Examples | Figures | Questions |
|---|---|---|---|---|---|---|
| 1. | Graphs | 32 | 21 | 29 | 37 | 24 |
| 2. | Representation of a Graph and Graph Isomorphism | 24 | 2 | 15 | 16 | 18 |
| 3. | Paths and Circuits | 36 | 26 | 27 | 33 | 15 |
| 4. | Shortest Path Algorithms | 34 | 8 | 7 | 37 | 16 |
| 5. | Trees | 46 | 8 | 36 | 70 | 20 |
| 6. | Planar Graphs and Coloring | 30 | 5 | 19 | 40 | 18 |
| 7. | Networks, Flows and Cuts | 18 | 7 | 7 | 11 | 7 |
| 8. | Applications of Graphs | 20 | 4 | 20 | 24 | — |
| 9. | Important Questions – Solved | 42 | — | — | 87 | 53 |
| 10. | Miscellaneous Questions | 12 | — | — | 39 | 64 |
| 11. | Question Papers | 16 | — | — | 40 | 26 |
| | **Total** | | **81** | **160** | **434** | **261** |

# Contents

# Syllabus

## University of Delhi

***Undergraduate Curriculum Framework (UGCF) – 2022***

Based On

***National Education Policy 2020***

**B.Sc. (Hons.) Mathematics - III Semester***

**Discipline Specific Elective Course-I**

**Graph Theory**

**Unit 1: Graphs, Paths and Circuits**

Definition, Examples and Basic Properties of Graphs, Subgraphs, Pseudographs, Complete Graphs, Bipartite Graphs, Isomorphism of Graphs, Paths and Circuits, Connected Graphs, Eulerian Circuits, Hamiltonian Cycles, Adjacency Matrix, Weighted Graph, Travelling Salesman Problem, Shortest Path, Dijkstra's Algorithm.

**Unit 2: Applications of Paths and Circuits, Trees**

Applications of Path and Circuits. The Chinese Postman Problem, Digraphs, Bellman - Ford Algorithm, Tournaments, Scheduling Problem, Trees, Properties of Trees, Spanning Trees, Minimum Spanning Tree Algorithms.

**Unit 3: Connectivity and Graph Coloring, Planar Graphs**

Cut - Vertices, Blocks and Their Characterization, Connectivity, and Edge Connectivity, Planar Graphs, Euler's Formula, Kurotowski Theorem, Graph Coloring and Applications, Matchings, Halls Theorem, Independent Sets and Covers.

## University of Delhi

**Bachelor in Multidisciplinary Courses of Study with**
**3 Core Courses (II Semester)**
**GE-2: Introduction to Graph Theory**

**Unit 1: Graphs, Paths and Circuits**

Definition, Examples and Basic Properties of Graphs, Subgraphs, Pseudographs,

* As per annexure – 4.01.46

Complete Graphs, Bipartite Graphs, Isomorphism of Graphs, Paths and Circuits, Eulerian Circuits, Hamiltonian Cycles, Adjacency Matrix, Weighted Graph, Travelling Salesman Problem; Shortest Path, Dijkstra's Algorithm.

**Unit 2: Applications of Paths and Circuits, and Tree Algorithms**

The Chinese Postman Problem, Digraphs, Bellman-Ford Algorithm, Tournaments, Scheduling Problems, Trees and their Properties, Spanning Trees, Minimum Spanning Tree Algorithms, Acyclic Graphs and Bellman's Algorithm.

**Unit 3: Planar Graphs, Colorings and Matchings**

Planar Graphs, Euler's formula, Kuratowski Theorem, Coloring Graphs, Applications of Coloring Graphs, Circuit Testing and Facilities Design, Matchings, Hall's Theorem.

## Chaudhary Charan Singh University, Meerut

### B.A./B.Sc. III (Semester-V) Paper-II
### Graph Theory & Discrete Mathematics
### Part A - Graph Theory

**Unit 1:**

Introduction to Graphs, Basic Properties of Graphs, Simple Graph, Multi Graph, Graph Terminology, Representation of Graphs, Bipartite, Regular, Planar and Connected Graphs, Connected Components in a Graph, Euler Graphs, Directed, Undirected, Multi-Graph, Mixed Graph.

**Unit 2:**

Walk and Unilateral Components, Unicursal Graph, Hamiltonian Path and Circuits, Graph Coloring, Chromatics Number, Isomorphism and Homomorphism of Graphs, Incidence Relation and Degree of the Graph.

**Unit 3:**

Operation of Graph Circuit, Path and Circuits, Eulerian Circuits, Hamiltonian Path and Cycles, Adjacency Matrix, Weighted Graph, Travelling Salesman Problem, Shortest Path, Dijkstra's Algorithm.

**Unit 4:**

Tree, Binary and Spanning Trees, Coloring, Color Problems, Vertex coloring and Important Properties.

### Part B - Discrete Mathematics

**Unit 5:**

**Propositional Logic-** Proposition Logic, Basic Logic, Logical Connectives, Truth Tables, Tautologies, Contradiction, Normal Forms (Conjunctive and Disjunctive), Modus Ponens and Modus Tollens, Validity, Predicate Logic, Universal and Existential Quantification, Proof by Implication, Converse, Inverse Contrapositive, Contradiction, Direct Proof by using Truth Table.

**Unit 6:**

**Relation-** Definition, Types of Relation, Domain and Range of a Relation, Pictorial Representation of Relation, Properties of Relation, Partial Ordering Relation. Representation of POSETS using Hasse Diagram, Chains, Maximal and Minimal Point. Glb, Lub, Lattices and Algebraic System, Basic Properties, Sublattices.

**Unit 7:**

Boolean Algebra- Basic Definitions, Sum of Products and Products of Sums, Boolean Functions, Disjunctive Normal Form, Complete Disjunctive Normal Form, Conjugate Normal Form, Logic Circuits, Logic Networks, Design of Circuits from Given Properties, Logic Gates, and Karnaugh Maps.

**Unit 8:**

Combinatorics- Inclusion- Exclusion, Recurrence Relations ($n^{th}$ Order Recurrence Relation with Constant Coefficients, Homogeneous Recurrence Relations, Inhomogeneous Recurrence Relations), Generating Function (Closed Form Expression, Properties of G.F., Solution of Recurrence Relations using G.F. Solution of Combinatorial Problem using G.F.

## UNIVERSITY OF MADRAS

### B.Sc. In Mathematics

### Elective-II / III: Graph Theory

**Unit 1:**

Graphs and Subgraphs: Introduction- Definition and Examples, Degrees, Sub Graphs, Isomorphism, Independent Sets and Coverings, Intersection Graphs and Line Graphs, Matrices, Operations on Graphs.

**Unit 2:**

Degree Sequences and Connectedness: Degree Sequences and Graphic Sequences – Simple Problems. Walks, Trails, Paths, Connectedness and Components, Blocks, Connectivity – Simple Problems.

**Unit 3:**

Eulerian and Hamiltonian Graphs.

**Unit 4:**

Trees : Characterisation of Trees, Centre of a Tree - Simple Problems. Planarity: Definition and Properties, Characterization of Planar Graphs.

**Unit 5:**

Directed Graphs: Definition and Basic Properties, Paths and Connections, Digraphs and Matrices, Tournaments.

# Glossary of Symbols

| | | |
|---|---|---|
| $\in$ | : | Belongs to |
| $\notin$ | : | Does not belong to |
| $\forall$ | : | For all |
| $\exists$ | : | There exists |
| $\Rightarrow$ | : | Implies |
| $\Leftrightarrow$ | : | Implies and is implied by |
| iff | : | If and only if |
| $\wedge$ | : | Meet (and) |
| $\vee$ | : | Join (or) |
| $a\|b$ | : | $a$ divides $b$ |
| $\varphi$ | : | Empty set |
| $\cong$ | : | Is isomorphic to |
| N | : | Set of natural numbers |
| Z | : | Set of integers |
| ■ | : | End of proof |
| $\nexists$ | : | Does not exist |
| $\sum$ | : | Sum |
| $\prod$ | : | Product |
| $\subseteq$ | : | Is a subset of |
| $\subset$ | : | Is a proper subset of |
| $\cup$ | : | Union |
| $\cap$ | : | Intersection |
| $\setminus$ | : | Set difference |

| | | |
|---|---|---|
| $A^c$ | : | The complement of $A$ |
| $\|A\|$ | : | The Cardinality of $A$ (if $A$ is finite, then it is number of elements in $A$) |
| $R$ | : | Set of real numbers |
| $C$ | : | Set of complex numbers |

## About the Book

This book, "Introduction to Graph Theory" is written for Undergraduate students of different disciplines/ courses across the Universities in India under NEP, CBCS and LOCF. This book has great value for readers who wish to go beyond the routine manipulations of formulas to solve standard problems, because it develops the ability to think deductively, analyze mathematical situations, and extend ideas to a new context. It covers both theoretical and applied approaches in a very simple manner. It provides ample amount of solved problems and lengthy lists of exercises, some easy and some challenging.

## Salient Features

- Application based approach will give motivation to the undergraduate students for interdisciplinary research.
- Examples and solved sample questions will provide different techniques used to crack a problem.
- Variety of applications discussed to visualize the relevance of graphs in different areas of mathematics.
- Exercises include some challenging questions for the readers which will enhance conceptual ability and give them an idea what type of real life problems can be approached with graphs.

## About the Authors

**Dr. Mamta Chaudhary** is an Associate Professor in the Department of Mathematics, Satyawati College. She brings 25 plus years of rich teaching experience in different courses in Delhi University and has taught Mathematical Analysis, Differential Equations, Algebra, Discrete Mathematics, Graph Theory and Metric Spaces at undergraduate level. She completed her doctorate in 2018 in Mathematical Programming from the University of Delhi and is still pursuing research work.

**Dr. Vani Sharma** is an Associate Professor in the Department of Mathematics, Satyawati College, University of Delhi. She has an experience of 23 years of teaching Undergraduate and Postgraduate students of University of Delhi. She is an alumna of Ramjas College and has been college topper for five years continuously. Dr. Vani has published several research papers in National and International journals. She is a lifetime member of National Research Group, Operational Research Society of India and International Research Group (Working Group of Generalized Convexity).

**Dr. Pooja Yadav** is working as an Assistant Professor in the Department of Mathematics, Kamala Nehru College, University of Delhi since 2010. In her 12 years experience, she has taught almost all the papers of Discrete Mathematics. She did her Ph.D in 2010 in Algebra from Indian Institute of Delhi (IITD). She was awarded scholarship by National Board of Higher Mathematics (NBHM). Currently, she is supervising the Ph.D. students as well.

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