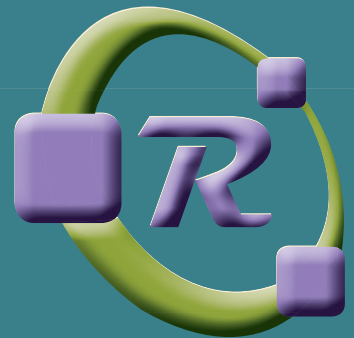
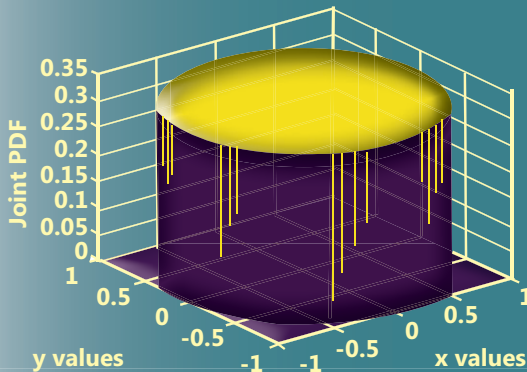


Probability Distributions AND Queueing Theory Using \mathcal{R} AND Octave



A Jiran Meitei

Sultan Chand & Sons

**PROBABILITY
DISTRIBUTIONS AND QUEUEING
THEORY USING
R AND OCTAVE**

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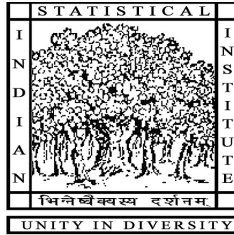
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My Family and Teachers

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Forward

I am delighted to introduce the book titled *Probability Distributions and Queueing Theory Using R* and *Octave* authored by Dr. A. Jiran Meitei. This book is an invaluable resource designed specifically for students pursuing disciplines such as statistics, mathematics, operational research, economics, commerce, and related fields at both undergraduate and postgraduate levels. The primary objective of the book is to present the complex subject matter in a simple and user-friendly manner, ensuring that students can easily grasp the concepts and apply them effectively. The book places significant emphasis on the practical use of open-source software tools, R and Octave, which are readily available for download. It also incorporates the use of MS Excel, including R programs for generating statistical tables, enabling students to seamlessly work with various probability distributions.

Comprising nine comprehensive chapters, the book covers a wide range of topics related to probability distributions and queueing theory. Two dedicated chapters delve into stochastic processes and queueing theory, providing in-depth discussions and valuable insights. Each chapter focuses on specific topics and includes appropriate examples to facilitate better understanding. Furthermore, exercises at the end of each chapter allow students to practice solving a variety of problems, reinforcing their comprehension and analytical skills.

Throughout the book, the author demonstrates the use of different R packages, providing students with a comprehensive understanding of these tools for statistical analysis and problem-solving. In summary, *Probability Distributions and Queueing Theory Using R and Octave* is a comprehensive and accessible guide that aims to equip students with a strong foundation in probability distributions and queueing theory. By emphasizing the practical application of R, Octave, and MS Excel, this book ensures that students can confidently apply their knowledge in real-world scenarios. Dr. Meitei's expertise and dedication shine through the content, making this book an invaluable resource for students and practitioners alike.

July 24, 2023


Shanta Laishram

P

Preface

The book titled '**Probability Distributions and Queueing Theory Using R and Octave**' is authored by **Dr. A. Jiran Meitei**, and aims to cater to students pursuing disciplines such as statistics, mathematics, operational research (OR), economics, commerce, and related fields at both undergraduate and postgraduate levels. The author's primary goal is to present the content in a simple and user-friendly manner.

The book extensively focuses on demonstrating the practical utilization of R and Octave, which are open-source software tools readily available for download, for working with various probability distributions. Additionally, the book incorporates the use of MS Excel for working with these distributions, including R programs for generating statistical tables.

Comprising nine chapters, the book covers a wide range of topics related to probability distributions and queueing theory. Two chapters are dedicated specifically to stochastic processes and queueing theory. Each chapter covers specific topics in detail and provides appropriate examples to facilitate better understanding. Moreover, exercises are included at the end of every chapter to allow students to practice solving a variety of problems.

Chapter 1 serves as an introduction, providing an overview of different types of random variables and their associated probabilities. Octave programs for plotting the Probability Mass Function (PMF), Joint PMF, and Probability Density Function (PDF) are provided to aid readers. **Chapter 2** delves into expectations and their properties, Moment Generating Function (MGF), cumulants, and other aspects related to random variables.

Chapter 3 focuses on discrete probability distributions, including Binomial, Poisson, Geometric, Negative Binomial, and Hypergeometric distributions. It provides detailed discussions on the properties and interrelationships of these distributions. **Chapters 4 and 5** cover important univariate continuous probability distributions. **Chapter 6** explains significant sampling distributions, including their properties and interrelationships. **Chapter 7** presents the fitting of various probability distributions, illustrating the use of the 'fitdistrplus' package in R for fitting univariate distributions. The last two chapters, **Chapters 8 and 9**, are dedicated to stochastic processes and queueing theory. The book also explains the functioning of the 'markovchain' package in R and Jensen MS Excel add-ins.

Overall, the book aims to provide students with a comprehensive understanding of probability distributions and queueing theory, emphasizing the practical application of R, Octave, and MS Excel in statistical analysis and problem-solving.

In expressing gratitude, the author acknowledges the support received from his family, friends, teachers, and colleagues, without whom completing this book would not have been possible. The author also acknowledges the contribution of his students, specifically **Mr. Ashish Singh**, who provided corrections and feedback on the chapters. Furthermore, the author extends appreciation to the teachers who imparted knowledge during his college days and acknowledges the help received from colleagues in developing the content of the book.

Finally, the author extends his gratitude to the esteemed publisher, **Sultan Chand and Sons**, for accepting to publish this work without delay.

A Jiran Meitei

Packages Used

R

<i>Package Name</i>	<i>Description</i>
---------------------	--------------------

fastGraph	Fast Drawing and Shading of Graphs of Statistical Distributions
fitdistrplus	Help to Fit of a Parametric Distribution to Non-Censored or Censored Data
diagram	Visualising simple graphs, flowcharts and webs
LaplacesDemon	Complete Environment for Bayesian Inference
fMultivar	Modeling of Multivariate financial return
markovchain	Package for Easily Handling Discrete Markov Chains in R
MASS	Support Functions and Datasets for Venables and Ripley's MASS

After installation, we need to load the library of the package before starting to use it, for example:

```
install.packages("fastGraph")  
library(fastGraph)
```

Octave

<i>Package Name</i>	<i>Description</i>
---------------------	--------------------

statistics	A collection of functions for statistical analysis including the probability distributions.
------------	---

We need to run **pkg load statistics** in the command window before start using this package.

MS Excel

<i>Package Name</i>	<i>Description</i>
---------------------	--------------------

queue.xla	This is a Jensen Excel Add-ins, in the Queuing Add-in and computes steady-state measures associated with Poisson queuing models, non-Markovian queues, and networks of queues. Both open and closed Markovian queues are modeled. The program also simulates multiple channel queues using two methods, a discrete next-event simulation, and an entity simulation. (https://utw11041.utweb.utexas.edu/ORMM/excel/queue.html)
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SP

Snapshot of the Book

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About the Book

This book enables learners of Statistics, Mathematics, Operational Research, Economics, Commerce, and related disciplines to understand univariate probability distributions using R and Octave. The explanations and presentations in the book are simple and user-friendly. The book also focuses on Stochastic Process and Queueing Theory. Ample examples and exercise questions supplement the discussions in each chapter. All the statistical tables in the book are generated using the R (R codes are included for readers). The extensive use of MS Excel for working with distributions is another added feature.

Salient Features

- The book covers all the major univariate probability distributions and the basics of stochastic processes along with the Markovian queueing systems.
- The New Educational Policy framework promotes the use of open-source softwares. R and Octave are two powerful open-source softwares which are freely accessible on the internet.
- Each distribution is explained using R, Octave and MS Excel.
- All chapters are supplemented with ample examples.
- The book's content will be helpful to students of statistics, mathematics, operational research, business economics, and management.

About the Author

Dr A Jiran Meitei is an Associate Professor in the Department of Mathematics at Maharaja Agrasen College, University of Delhi. Jiran has a postgraduate degree in Operational Research and a doctorate in Statistics. He has taught various disciplines of Mathematical Sciences for more than 21 years to, undergraduate and postgraduate students. He has several research publications in various national and international journals. His areas of interest include probability, stochastic processes, queueing theory, linear & non-linear programming, machine learning, and artificial intelligence.



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