Objective Statistics

- *For Under-Graduate and Post-Graduate Students of Statistics, Mathematics and Computer Science; I.S.S., CSIR (NET, JRF), I.I.T. JAM, S.S.C., R.B.I. and other Competitive Examinations*
- *Contains over 1,600 Objective Types Questions along with Solutions*

S.C. Gupta
Sanjeev Kumar Gupta
Ananya Gupta

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SULTAN CHAND & SONS®
Educational Publishers
New Delhi

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Educational Publishers
23, Daryaganj, New Delhi-110 002
Phones : 011-23281876, 23266105, 41625022 *(Showroom & Shop)*
011-23247051, 40234454 *(Office)*
E-Mail : sultanchand74@yahoo.com; info@sultanchandandsons.com
Fax : 011-23266357; Website : www.sultanchandandsons.com
ISBN : 978-93-91820-59-6 (TC-1272)
Price : ₹ 795.00
First Edition: 2024

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Author's Acknowledgement: The writing of a Textbook always involves creation of a huge debt towards innumerable authors and publications. We owe our gratitude to all of them. We acknowledge our indebtedness in extensive footnotes throughout the book. If, for any reason, any acknowledgement has been left out we beg to be excused. We assure to carry out correction in the subsequent edition, as and when it is known.

Printed at: Himani Print Solution, Badarpur, New Delhi-110044

P

Preface

Most institutions / organisations in India are conducting examinations under 'Multiple Choice Tests', which appears to be a very effective technique. Most of the students, who study Statistics used to work through short answers for problem solving and often, are not prepared for Multiple Choice Questions, (M.C.Q.'s). This book is planned to cover the syllabus of most of the examinations conducted in India and gives the students a large number of questions / problems for their learning and practice. This book will also be useful for students appearing in the examinations in the field of Statistics with the rigour of competitive examinations. The book covers the topics of Mathematical Statistics, generally asked under the various examinations, *viz*.,

1. Indian Statistical Services (I.S.S.) ;
2. Staff Selection Commission (S.S.C.) ;
3. Junior Statistical Officers (JSO) ;
4. Joint Admission Test for Masters. (JAM) conducted by Indian Institute of Technology (I.I.T.);
5. CSIR (J.R.F. / NET) for Subject Mathematical Sciences;
6. Reserve Bank of India, (RBI) ;
7. Various Entrance Examinations, for admission in M.Sc. and Ph.D. courses in Indian Universities;
8. 'Common University Entrance Test' (CUET) for admission in Central Universities across India.

The M.C.Q.'s asked in the examination are generally framed:

(*a*) To test the understanding and applications of the '*definitions*' of various terms used in the theory.

(*b*) To test the understanding and applications of the '*Statements of Theorems*', including the conditions (whether necessary or sufficient or both), under which they are valid.

(*c*) To test the accuracy of the numerical computations and use of '*Statistical Tables*'.

The best way to solve any problem lies in extracting the maximum information (explicit or implicit or both), from the language of the problem and then use it systematically in proper sequence to solve the problem.

The present book is a modest though determined bid to meet the requirements of the students of Mathematical Statistics at Degree, Honours and Post-graduate levels, and those preparing for the competitive examinations listed in the first paragraph.

The book is designed after carefully analysing the syllabus and papers of various competitive examinations and the whole relevant theory is written in such a way that the students may easily and effectively understand the key / basic concepts of mathematical statistics and use them intelligently in

solving the problems in minimum possible time. The problems asked in the competitive examination are innovative, some of which are really tough and difficult and often require some 'out of the box' solutions. The book contains 13 Chapters, containing 34 Exercise Sets and 1607 M.C.Q.'s or Problems.

(A) While keeping in mind the varied requirements at different levels, an attempt has been made to start each Chapter with a brief summary of the important definitions, theorems and the main results on each topic, thus covering all the basic concepts of that topic.

(B) This is followed by a number of "Exercise Sets", containing M.C.Q.'s on various topics covered in the Chapter. A variety of M.C.Q.'s have been framed on each topic keeping in mind the points (a), (b) and (c) in paragraph 2.

In addition, a number of M.C.Q.'s selected from various competitive examination papers have also been included in the Exercise Sets. An attempt has been made to start with simple problems and then harder and advanced problems have been given.

(C) Each 'Exercise Set', is followed by the 'Answer Sheet'.

(D) Each 'Answer Sheet', is followed by 'Solutions To Exercise Set', in which the solution to each question in the Exercise Set has been provided. The main emphasis is on the understanding of the definitions / theorems and their applications, and expose the readers to different techniques of tackling the problem and enable them to have a better and thoughtful understanding of the basic concepts of the subject.

At many places "Explanatory Remarks", and 'Alternative Methods', have been given to widen the readers' horizon. This will enable the readers to have a proper appreciation of the subject matter and fortify their confidence in the understanding and application of various methods. A careful study of the '*Solutions of the Problems*', is bound to motivate and stimulate the readers to work out any other problem independently and instill confidence in them.

Note: After carefully studying the theory, the readers are advised to try to solve the problems themselves without looking at the solutions. If they fail to solve them, only then they should look at the solutions.

There are few books by other publishers on M.C.Q.'s in Statistics but they are very elementary and hardly cater to the requirements of the students preparing for the examinations listed in paragraph 1. Most of them have not given answers to the exercise sets. A special feature of our book is that we have provided 'Answer Sheets', as well as 'Solutions', to each problem in the Exercise Sets. Thus, our book is quite exhaustive and caters to the requirements of a very large number of students.

We take this opportunity to express our sincere gratitude and thanks to Prof. Kamal Nain Kapoor, Hindu College, Delhi University, Delhi for his valuable help and suggestions in the preparation of this book.

Last, but not the least, we express our deep sense of thanks and gratitude to our publishers Sultan Chand & Sons, Darya Ganj, Delhi for their untiring efforts, unfailing courtesy and co-operation in bringing out the book in such an elegant form.

Every effort has been made to eliminate the mistakes and printing errors, though some might have crept in inadvertently. We shall be obliged if any such errors are brought to our notice. Valuable suggestions and fruitful criticism for the improvement of the book will be highly appreciated and duly incorporated in the subsequent edition.

S.C. Gupta

Sanjeev Kumar Gupta

Ananya Gupta

4th February, 2024

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About the Book

Most of the Institutions / Organisations in India are conducting examinations under "Multiple Choice" Tests for admissions or jobs in their Institutions or Organisations. The book *Objective Statistics* is a comprehensive book in statistics based on Multiple Choice Questions (MCQs) and is intended for the students preparing for the competitive examinations conducted by Indian Statistical Service (I.S.S.); Staff Selection Commission (S.S.C.); Junior Statistical Officers (JSO); IIT Joint Admission Test for Masters (JAM); CSIR (NET; JRF) for subject Mathematical Sciences and many other Competitive Examinations. This book is a modest, though determined bid to meet the requirements of the students of Mathematical Statistics at Undergraduate (Pass and Honours) and Postgraduate level and those preparing for the competitive examinations mentioned above. The book contains 13 Chapters, 34 Exercise Sets and 1,607 Multiple Choice Questions. A special feature of the book is that at the end of each Exercise Set the 'Answer Sheet' as well as the 'Solutions' to each question in the Exercise Set have been given.

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S.C. Gupta, (born 1937) has a brilliant academic record with Honours in Mathematics (1957), from Punjab University, Post-Graduation in Mathematical Statistics (1959) and Mathematics (1966), both from University of Delhi, and M.S. in Statistics (1971-72) from the University of North Carolina, Chapel Hill (U.S.A.). Mr. Gupta started his teaching career since July 1960 as a lecturer in the Department of Statistics, Meerut College, Meerut and then as Associate Lecturer in Statistics at I.I.T. Kharagpur. He was an eminent and dedicated faculty member of Hindu College, University of Delhi, since September 1963 and retired as Associate Professor of Statistics in November 2002. Mr. Gupta was a Visiting Scientist at Clark Atlanta University, Atlanta GA, (U.S.A.) in 2002. During his teaching career spanning over four decades, Mr. Gupta has taught numerous Statistics courses like Statistical Methodology, Theory of Probability, Probability Distributions, Applied Statistics, Sample Surveys and Sampling Techniques, Design of Experiments, Theory of Estimation and Testing of Hypothesis, *etc.* at the Under-graduate (Honours) level. He has also been associated with post-graduate teaching at the Department of Statistics, University of Delhi, for over 15 years. A well-established author of international repute, he has written books on *Mathematical Statistics, Fundamentals of Statistics, Applied Statistics, Business Statistics, Engineering Mathematics and Matrices* & *Wonderland of Numbers* which have been prescribed as text-books/reference books in most of the universities. Reading, writing and teaching are still his main passion.

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ISBN 978-93-91820-59-6

TC-1272