A TEXTBOOK ON

# MATHEMATICAL ANALYSIS

Prof. (Dr.) Chaitanya Kumar • Prof. (Dr.) Sarla Devi Bharadwaj
Dr. Sudha Gupta • Dr. Ritu Aggarwal

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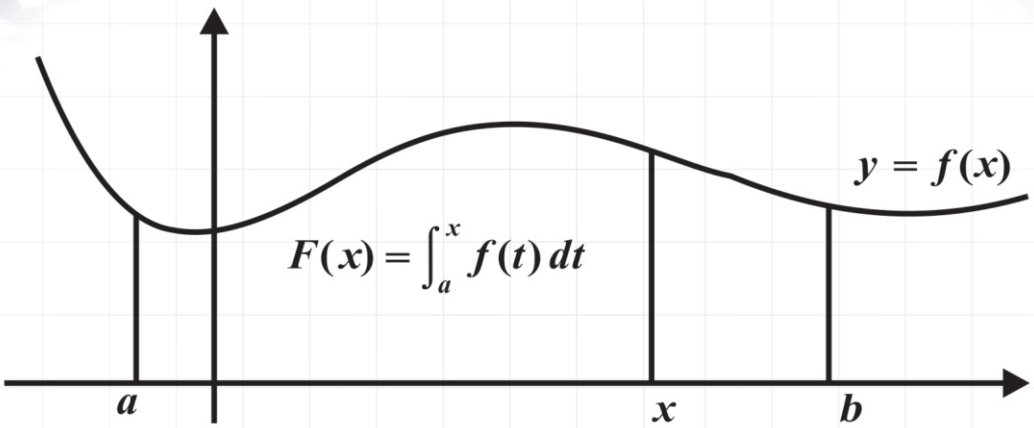


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# *A Textbook on*
# Mathematical Analysis

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Sultan Chand & Sons®
*Educational Publishers*
*New Delhi*

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*Educational Publishers*
23, Daryaganj, New Delhi-110 002
Phones : 011-23281876, 23266105, 41625022 *(Showroom & Shop)*
011-23247051, 40234454 *(Office)*
E-Mail : sultanchand74@yahoo.com; info@sultanchandandsons.com
Fax : 011-23266357; Website : www.sultanchandandsons.com

**ISBN : 978-93-49290-01-3 (TC-1345)**

***Price*** **: ₹ 250.00**

**First Edition:** 2025

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*Printed at:* Himani Print Solution, Badarpur, New Delhi-110044

# PREFACE

Computers can quickly substitute numbers in mathematical expressions and find their values, but the logical reasoning and mathematics behind these expressions must be provided by someone proficient in mathematics, especially in mathematical analysis. A solid understanding of mathematical analysis is essential for developing any branch of mathematics, whether it be applied mathematics, differential equations, computers, or statistics.

The authors are pleased to present the book entitled "*A Textbook on Mathematical Analysis*". Some of the notable features of the book include motivation for the theory and its application considering importance of the subject and level of the student. The content is well known and can be found in various books. The purpose of the present book is to prepare the topics in accordance with the Undergraduate Curriculum Framework (UGCF 2022) followed by central universities of India, including University of Delhi under the National Education Policy 2020. The present book will be useful for the students of B.Sc. (Hons) Mathematics (DSC 10) Semester V, GE 5 (Hons) courses Semester V (other than mathematics), B.A.(Prog) Semester VI with mathematics as major.

It is also instrumental for preparation of competitive examinations. The students of open and distance education courses will also find the book very beneficial. The topics have been presented in a simple, clear and coherent style with the help of diagrams and graphs where required, along with a numerous examples and questions, including those from recent university examinations.

The book comprises of seven chapters. Each chapter clearly explains the concepts and provides motivation about the topic. Various examples are given to provide the students an integral view of the theory and its applications. The first chapter is introductory, covering the knowledge of functions with the diagrams of functions which are needed for the following chapters. Second and third chapters discuss the limits and continuity of functions to lay foundation for subsequent chapters. Chapter four covers uniform continuity with plenty of examples so that students can understand the concept thoroughly. Chapter five focuses on Riemann integration and the topic is explained with various examples and diagrams. Chapter six and seven are for uniform convergence and power series. The authors do not

claim the originality of the topic. The purpose of the present book is to give complete syllabus at one place particularly related to mathematical analysis papers.

The authors sincerely welcome constructive suggestions from students and teachers.

Authors are indebted to Prof. Rajiv Chopra (principal, Delhi College of Arts and Commerce), Prof. Sadanand Prasad (OSD, Dr. Bhim Rao Ambedkar College), Prof. Pratyush Vatsala (Principal, Lakshmi Bai College), Prof. Subhash Kumar Singh (Principal, Satyawati college(M)), Prof. Tarun Das (HOD, Department of Mathematics), Prof C.S. Lalitha, Prof. Shiv Kumar Sahdev, Prof. Gopal Datt, Prof. Chaman Singh, Prof. Pankaj Garg, Prof. Ratnesh, Prof. Dhanpal, Prof. Sudha Arora, Prof. Monika, Prof. Dinesh Khattar.

Finally and above all authors offers their biggest appreciation to their family members who have shown unbounded patience throughout the period of writing the text.

***Prof. Chaitanya Kumar***
***Prof. Sarla Bhardwaj***
***Dr. Sudha Gupta***
***Dr. Ritu Aggarwal***

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# SNAPSHOT OF THE BOOK

| *No.* | *Chapter Name* | *Example* | *Definition* | *Exercise with Sol.* | *Theorem* | *Figure* | *Self Ass. Ex.* | *Lemma* | *Corollary* |
|---|---|---|---|---|---|---|---|---|---|
| 1. | Functions | 6 | 9 | 5 | – | 12 | 10 | – | – |
| 2. | Limits of Functions | 26 | 11 | 12 | 12 | 7 | 5 | – | – |
| 3. | Continuous Functions | 25 | 13 | 12 | 13 | 6 | 5 | – | – |
| 4. | Uniform Continuity | 9 | 1 | 8 | 7 | 2 | 8 | – | – |
| 5. | Riemann integration | 22 | 7 | 19 | 24 | 23 | 9 | 2 | 2 |
| 6. | Uniform Convergence of Functions | 15 | 3 | 6 | 10 | – | 10 | – | – |
| 7. | Power Series | 25 | 4 | 11 | 5 | – | 7 | – | 5 |
| Sample Paper-1 | | – | – | – | – | – | – | – | – |
| Sample Paper-2 | | – | – | – | – | – | – | – | – |
| **Total** | | **128** | **48** | **73** | **71** | **50** | **54** | **2** | **7** |

# A Textbook on
# Fundamentals of Calculus

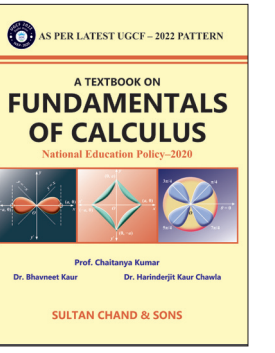
*National Education Policy – 2020*

Prof. Chaitanya Kumar

Dr. Bhavneet Kaur • Dr. Harinderjit Kaur Chawla

## About the Book

This book has been designed in accordance with the Undergraduate Curriculum Framework-2022 followed by the Central Universities of India including University of Delhi under the National Education Policy (NEP)-2020. Keeping in mind the need to uphold students' interest in the subject, vivid explanation of concepts as well as explanatory illustrations followed by exercises have been included. The book is exclusively designed to help and guide the students of Mathematics DSC-5 B.Sc. (Hons.) Mathematics; GE-1(i) B.Sc./B.A. (Hons.) (Other than Mathematics); Discipline A-1 and GE-1(i) Bachelor in Multidisciplinary Courses. It is also useful for B.Tech. students of various Universities and for preparation of competitive examinations. The students of open and distance education courses will also find the book very beneficial.




## Salient Features

- An all-encompassing and self-sufficient textbook for UGCF-2022 based on NEP-2020.
- Written in a lucid and simple language.
- Written with a view to present a qualitative understanding of the subject.
- Comprehensive step-by-step explanation for easier understanding of the subject.
- Many solved examples and unsolved problems have been drawn from recent examination papers of Universities.
- Answers to all the problems in each exercise have been provided immediately after the exercise for the convenience of the reader.
- Recent Delhi University Question Papers with Solutions have been included for ample practice.

## Contents

- Limits, Continuity and Differentiability
- Successive Differentiation
- Partial Differentiation
- Tangents and Normals
- Asymptotes and Singular Points
- Maxima and Minima
- Curve Tracing
- Mean Value Theorems
- Indeterminate Forms
- Reduction Formulae
- *University Question Papers with Answers*

Edition 1st, 2022; Pages: *xvi* + 320; Size 185 × 240 mm; ISBN: 978-9-391820-01-5 (TC-1269)

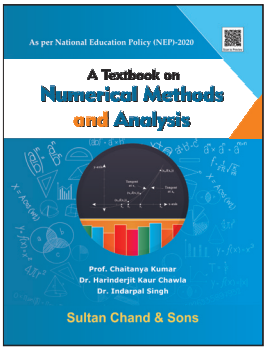
# A Textbook on
# Numerical Methods and Analysis

*National Education Policy – 2020*

Prof. Chaitanya Kumar

Dr. Indarpal Singh • Dr. Harinderjit Kaur Chawla

## About the Book




This book has been designed in accordance with the Undergraduate Curriculum Framework-2022, followed by the Central Universities of India, including University of Delhi under the National Education Policy (NEP)-2020. It is exclusively crafted to cater to the interest of students of B.Sc. (Hons.) Mathematics DSC-12 Semester-IV; GE-5(I) B.Sc./B.A. (Hons.) (other than Mathematics) Semester-V; B.Sc. (Physical Sciences/ Mathematical Sciences) DSE-1(b) Semester-V; GE-5(I) Semester-V and DSC-13 Semester-VII Bachelor in Multidisciplinary courses with 2 core and 3 core disciplines; B.Sc.(Hons.) Operational Research DSE-1(b) Semester-V and B.Sc. (Hons.) Statistics DSE-09 Semester-V. It is also useful for B.Tech students of various Universities and for the preparation of competitive examinations. The students of open and distance education courses will also find the book very beneficial.

## Salient Features

- An all-encompassing and self-sufficient textbook for UGCF-2022 based on NEP-2020.
- Written in lucid and simple language.
- Written with a view to present a qualitative understanding of the subject.
- Comprehensive step-by-step explanations for easier understanding of the subject.
- Many solved examples and unsolved problems have been drawn from recent examination papers of Universities.
- Answers to all problems in each exercise have been provided immediately after the exercise for the convenience of the reader.
- Recent Delhi University Question Papers with Solutions have been included for ample practice.

## Contents

- Basic Elements of Numerical Methods and Error Analysis
- Methods for Solving Algebraic and Transcendental Equations
- Techniques to Solve System of Linear Algebraic Equations
- Finite Differences and Interpolation
- Numerical Differentiation
- Numerical Integration
- Numerical Solution of Ordinary Differential Equations
- Practicals in Numerical Analysis
- *University Question Paper-2021*
- *University Question Paper-2022*
- *University Question Paper-2023*

Edition 1st, 2024; Pages: *xii* + 236; Size 185 × 240 mm; ISBN: 978-93-91820-83-1 (TC-1313)

# A Textbook on
# Differential Equations and Applications

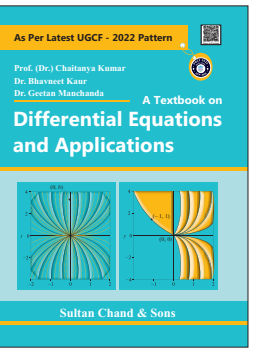
*As per Latest UGCF - 2022 Pattern*

Prof. Chaitanya Kumar

Dr. Bhavneet Kaur • Dr. Geetan Manchanda

## About the Book

This book has been designed in accordance with the Undergraduate Curriculum Framework-2022 followed by the Central Universities of India including University of Delhi under the National Education Policy (NEP)-2020. Keeping in mind the need to uphold students' interest in the subject, vivid explanation of concepts as well as explanatory illustrations followed by exercises have been included. The book is exclusively designed to help and guide the students of Mathematics DSC-6 B.Sc. (Hons.) Mathematics Semester-II; GE-3(I) all Honours Courses (Other than Mathematics); Discipline A-3 (3-Core & 2-Core). It is also useful for B.Tech. students of various Universities and for preparation of competitive examinations. The students of open and distance education courses will also find the book very beneficial.




## Salient Features

- An all-encompassing and self-sufficient textbook for UGCF-2022 based on NEP-2020.
- Written in a lucid and simple language.
- Written with a view to present a qualitative understanding of the subject.
- Comprehensive step-by-step explanation for easier understanding of the subject.
- Many solved examples and unsolved problems have been drawn from recent examination papers of Universities.
- Practicals using Softwares (Mathematica, Maxima & Octave) are included for better understanding of the subject.
- Recent Delhi University Question Papers with Solutions have been included for ample practice.

## Contents

- Preliminaries
- Ordinary Differential Equations of Order One and First Degree
- Differential Equations of First Order but Not to First Degree
- The Wronskian and its Properties
- Linear Differential Equations with Constant Coefficients
- Homogeneous Linear Differential Equations
- Method of Variation of Parameters and Undetermined Coefficients
- Simultaneous Linear Differential Equations
- Total Differential Equations
- Compartmental Models
- Models of Single Population
- Interacting Population and Phase-Plane Analysis
- Practicals in Differential Equations
    - Mathematica Software
    - Maxima Software
    - Octave Software
- University Question Papers (2018 – 2022)

Edition 1st, 2023; Pages: *xvi* + 520; Size 185 × 240 mm; ISBN: 978-93-19820-32-9 (TC- 1289)

## About the Book

This book has been designed in accordance with the undergraduate curriculum framework-2022 followed by the central universities of India including University of Delhi under the National Education Policy (NEP)-2020. Keeping in mind the need to uphold students' interest in the subject, vivid explanations of concepts as well as explanatory illustrations followed by exercises have been included. The book is exclusively designed to help and guide the students of B.Sc. (Hons.) Mathematics (DSC-10) semester V, GE-5 (all Honours courses other than Mathematics), B.A. (Prog.) Semester VI with mathematics as major. It is also useful for B.Tech students of various universities and for the students preparing for competitive examinations. The students of open and distance education courses will also find the book very beneficial.

## Salient Features

- Written in a simple and easy language to understand, as if a teacher is explaining it in a classroom.
- Comprehensive step by step explanation of concepts, with the help of examples and illustrations.
- Each chapter begins by learning objectives, to set up the context for what the reader can expect to learn from it.
- Solutions to all exercise problems are given in the end of the cheater.
- Practice sample papers are included to help students and teachers.

## About the Authors

**Prof. (Dr.) Chaitanya Kumar** has a brilliant academic record of teaching of more than four decades at Delhi College of Arts and Commerce, University of Delhi. He earned his M.Phil and Doctorate degree from the University of Delhi under the guidance of esteemed Prof. D.L. Jain. He has published multiple research papers in the area of Boundary Value problems and Integral equations in reputed international journal. He has been a research supervisor at postgraduate and higher levels. He has authored more than a dozen mathematics books.

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ISBN 978-93-49290-01-3
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