Calculus Using MATHEMATICA

Pragati Gautam Swapnil Verma



Sultan Chand & Sons

Calculus using Mathematica

Calculus using Mathematica

Dr. Pragati Gautam

Ms. Swapnil Verma



SULTAN CHAND & SONS[®]

Educational Publishers New Delhi

SULTAN CHAND & SONS®

23, Daryaganj, New Delhi-110002

Phone: 011-23281876, 23266105, 23277843 (Showroom & Shop) 011-40234454, 23247051 (office)

E-mail: sultanchand74@yahoo.com; info@sultanchandandsons.com Fax: 011-23266357; Website: www.sultanchandandsons.com

1st Edition: 2023

ISBN: 978-93-91820-26-8 (TC-1264)

Price: ₹ 250

EVERY GENUINE COPY OF THIS BOOK HAS A HOLOGRAM



In our endeavour to protect you against counterfeit/fake books, we have pasted a copper hologram over the cover of this book. The hologram displays the full visual image, unique 3D multi-level, multicolour effects of our logo from different angles when tilted or properly illuminated under a single light source, such as 3D depth effect, kinetic effect, pearl effect, gradient effect, trailing effect, emboss effect, glitter effect, randomly sparking tiny dots, micro text, laser numbering, etc.

A fake hologram does not display all these effects.

Always ask the bookseller to put his stamp on the first page of this book.

All Rights Reserved: No part of this book, including its style and presentation, may be reproduced, stored in a retrieval system, or transmitted in any form or by any means–electronic, mechanical, photocopying, recording or otherwise without the prior written consent of the Publishers. Exclusive publication, promotion and distribution rights reserved with the Publishers.

Warning : The doing of an unauthorised act in relation to a copyright work may result in both civil claim for damages and criminal prosecution.

Special Note : Photocopy or Xeroxing of educational books without the written permission of Publishers is illegal and against Copyright Act. Buying and selling of pirated books is a criminal offence. Publication of key to this is strictly prohibited.

General: While every effort has been made to present authentic information and avoid errors, the author and the publishers are not responsible for the consequences of any action taken on the basis of this book.

Limits of Liability/Disclaimer of Warranty : The publisher and the author make no representation or warranties with respect to the accuracy or completeness of the contents of this work and specifically disclaim all warranties, including without limitation warranties or fitness for a particular purpose. No warranty may be created or extended by sales or promotional materials. The advice and strategies contained herein may not be suitable for every situation. This work is sold with the understanding that the publisher is not engaged in rendering legal, accounting, or other professional services. If professional assistance is required, the services of a competent professional person should be sought. Neither the publisher nor the author shall be liable for damage arising herefrom.

Disclaimer : The publisher have taken all care to ensure highest standard of quality as regards typesetting, proofreading, accuracy of textual material, printing and binding. However, they accept no responsibility for any loss occasioned as a result of any misprint or mistake found in this publication.

Author's Acknowledgement : The writing of a Textbook always involves creation of a huge debt towards innumerable author's and publications. We owe our gratitude to all of them. We acknowledge our indebtedness in extensive footnotes throughout the book. If, for any reason, any acknowledgement has been left out we beg to be excused. We assure to carry out correction in the subsequent edition, as and when it is known.

Printed at : Himani Print Solution, Badarpur, New Delhi-110044.

Dr. Shiv Raj Singh Professor & Head



Department of Mathematics Ch. Charan Singh University Meerut-250004 (U.P) (INDIA).

Foreword

It is imperative to have a deep understanding of Mathematics in order to teach it in Higher Education. The most effective mathematics teachers think about and beyond the content, they teach, seeking explanations and correcting mistakes. Today's education system requires both theoretical and practical concepts. Mathematica software is used extensively in this book, which is written by the author for practical applications.

Calculus using Mathematica is a self-contained and unique introduction to the basics of Calculus using Mathematica software. In this book, the authors have presented topics emphasizing the development of calculus and its applications by using the software. Due importance is given to computational techniques involved in understanding the subject. This book is a product of their rich experience of teaching this subject to the students of Delhi university.

I really found the book interesting as throughout the book the authors highlight the fact that calculus is actually a foundation for several concepts and results that are taught in high school and accepted in the abstract. The book provides the visualisation of trigonometric, algebraic, exponential, and parametric curves, polar equations, and complex numbers. This book is written in simple language with a lot of examples to explain the concepts of various notions that will be interesting for students as well as Educators.

The only prerequisites for using this book are the topics that are covered in initial schooling; however, the reader should possess some mathematical maturity and an ability to understand and appreciate the graphs. This book is in a class by itself as it is the first book written on Mathematica covering the NEP syllabus. I hope that this book will become a primer for teachers, students and professionals.

Prof. (Dr.) Shiv Raj Singh

Prof. Santosh Kumar

Department of Mathematics College of Natural and Applied Sciences University of Dar es Salaam P. O. Box -35062, Tanzania Tel: +255 22 410 388 FAX: +255 22 2410514 Mobile: +255-765159750 E. Mail: drsengar2002@gmail.com

Foreword

It is my great pleasure to write a few lines about the effort of Dr. Pragati Gautam and Ms. Swapnil Verma in their book entitled "Calculus using Mathematica".

The authors have successfully managed to bring out a compact and complete text book for the students who needs knowledge of Calculus Using Mathematica. Calculus in its integral and differential form is incomplete without its computer applications. This book is designed as per the National Education Policy guidelines and provides information about Value Added Course (VAC) and skill enhancement program in which emphasis is laid on application of a subject. This particular book justifies the thought of education policy as it is based on learning of Calculus through Mathematica software.

The main motive of the software is to simplify the subject and to increase its visualization which is excellently done by the authors. The book is simple to understand as it starts from the very basics moving to the complex topic like parametric curves, polar equations etc. to name a few. The motive of the syllabus makers is to increase the application of the subject and to relate it to daily life. With this book, this intention is fulfilled as it explains each topic with graphs and examples. MCQ at the end of each chapter helps students to increase their rational thinking. This book will be useful for undergraduates/postgraduates/ engineering as well as medical students. It will enhance the mathematical concepts, for example hyperbolic functions can be viewed in a better manner via graphs. In the end, I wish all the success to the authors and congratulate them on their creative initiative.

(Prof. Santosh Kumar) Date: 16/12/2022

Preface

Learning a new concept is a difficult task by itself. The only thing that can make it less strenuous is by having access to the right resources. I would not go so far as to say that there are not any books discussing various topics in Mathematica in-depth. We can argue that the very contrary is true. Then you must wonder, why have we written a book on Mathematica? What has happened to the vast number of resources that we just mentioned?

While several excellent books are available to demystify complex Mathematica problems, these books have not been written while keeping the curriculum of NEP (2020) for B.Sc./B.A.(H) in Mathematics in mind. This book has been designed in accordance with the latest Undergraduate Curriculum Framework-2020 followed by the Central Universities of India including University of Delhi, C.C.S University, Meerut, etc under the National Education Policy (NEP)-2020. The book exclusively caters to the interests of students of mathematics B.Sc./B.A. (H)-Mathematics, B.Sc./B.A(H) – Other than Mathematics, Bachelor in Multidisciplinary courses and B.Tech students of various Universities. The book is extremely useful for preparation of competitive examinations. This book will act as a guide for learning Mathematica in open and distance education courses.

The importance of a book tailored specifically to your needs is only emphasized when students have just started college. The transition from high school to college is quite drastic, and often, students in their first semester find it challenging to rely just on readings for preparing for examinations or class tests. The present scenario is further compounded by the lack of relevant readings for Mathematica in most universities.

•••	Preface
x	Ne • •

Without access to a standard of notes, students are limited to resources provided in classes by teachers.

This book aims to reduce the discrepancy of knowledge that occurs when we adopt the previously discussed system. We hope that this book will not only serve to create a baseline knowledge for all students in Central Universities like the University of Delhi but also function as a guide for teachers on how each topic should be covered in class. By including numerous solved examples and exercise problems, we hope that this book will serve as a foundational resource for understanding key concepts related to Mathematica.

The contents of this book have been divided into thirteen chapters. It should be noted that the **first chapter** only delves into the various features provided by Mathematica. This chapter also builds our understanding of the resourcefulness of Mathematica as not just a scientific tool but also as an aid for complex tasks such as Machine Learning and Data Science. Finally, this chapter will also provide a tutorial on installing Mathematica. The second chapter will only focus on the various commands at our disposal for plotting graphs, animating graphs, and more. After building our understanding of the importance of Mathematica and the different ways we can use it for calculations, we dive into the diverse applications of the software in a Mathematical sense. For instance, the third chapter spends a considerable time discussing what we mean by limits, continuity, and differentiability in the program and the commands available to find them. In fact, in chapter six, we utilize the knowledge gained and explore the calculation of limits for hyperbolic functions. On the other hand, in chapters four, five and seven, we profoundly investigate the various plots Mathematica can build. These chapters are dedicated to topics centered around tracing parametric curves, polar equations, and equations like parabola, ellipse, and hyperbola. After examining the various two-dimensional plots at our disposal, we focus on plotting three-dimensional figures in Mathematica. Specifically, in **chapter twelve**, we investigate the different revolving surfaces that can be constructed about various axes. In the later chapters, like chapters nine and ten, we analyze the utility of Mathematica in the number systems by doing operations such as graphical representations of complex numbers or tracing of random real numbers. Another critical topic of interest for us is matrices. To ensure students have a fundamental understanding of the topic, we dedicate chapter eleven to the different use cases of matrices, such as the Cayley Hamilton theorem and solving the system of linear equations. The final chapters twelve and thirteen

Preface	•••
	xi

are spent understanding how Mathematica can be applied in problems related to sequences and series while covering important themes like Bolzano Weierstrass Theorem, Cauchy integral test, and D' Alembert ratio test.

Finally. none of this would have been possible without the constant support from our families, friends, and colleagues. However, the most considerable credit for our success goes to our students whom we have not only taught but also learned from throughout these years. This iterative teaching-learning process sharpens our conceptual clarity and prompts us to consider how we may convey complex topics most concisely and comprehensively possible. We also wish to say a special thanks to our students, **Ms. Medhavi Darshan and Ms. Komal Negi**, for their help in developing the content of this book. We express profound appreciation towards **Prof. S.R. Singh, Head of the Mathematics Department at C.C.S University**. His constant support has been our foundational pillar in creating Calculus using Mathematica particularly designed for the C.C.S. University students.

We would like to acknowledge the motivation and valuable suggestions received from various distinguished teachers, in particular, **Prof. B.K. Das** (*Rtd. University of Delhi*), **Sr. Prof. C.S. Lalitha** (*South Campus, University of Delhi*), **Prof. Ruchi Das** (*Head of Mathematics Department, University of Delhi*), **Prof. Dinesh Khattar** (*Principal, Kirorimal College, University of Delhi*), **Prof. Ratikanta Panda** (*University of Delhi*), **Dr. Ranjana Jain** (*University of Delhi*) and **Dr. Izhar Uddin** (*Department of Mathematics, Jamia Millia Islamia*) without which the book would not be in this form.

Ultimately, without any delay, we must share our gratitude towards our esteemed publisher **Sultan Chand and Sons**, without whom you would not be holding this book!

At the crux of it all, while we aim to create a comprehensive Mathematica practical book for our students, we must admit that at the rate at which Mathematica capabilities keep on growing, we do run the risk of leaving some important findings. In the event this does occur, please do not hesitate in sharing your feedback in terms of suggestions, comments, or even criticisms with us. After all, we only hope to improve the book in the future!

15th Dec, 2022

Dr. Pragati Gautam Ms. Swapnil Verma

Contents C

1 Introduction: Mathematica Software		
1.1 Introduction	1	
1.2 Installing Mathematica Software	2	
1.3 Basic Arithmetic manipulations	3	
1.4 How to write commands in Mathematica	5	
1.5 Basic Math Assistant	6	
1.6 Built-In Constants in Mathematica	9	
1.7 Saving Work and Quitting	11	
1.8 Using "Help" in Mathematica	11	
1.9 How To Open Saved Notebooks	12	
1.10 Writing Text in Mathematica Notebook	12	
1.11 Insert Graphics in Mathematica	13	
1.12 Writing Assistant Palette	13	
1.13 Editing in Mathematica	14	
1.14 Converting Notebook in Other Formats	14	
1.15 Creating Slideshow	14	
1.16 Printing the Document	14	
1.17 Loading Packages	15	

2 Functions in Mathematica

2.1 Introduction

xiv

2.2 Tracing of functions	21
2.2.1 Tracing of functions by using Plot command	21
2.2.2 Tracing of functions by using Manipulate command	25
2.2.3 Tracing of function by using Animate command	28
2.3 Graphics in Mathematica	29

3 Limit, Continuity and Differentiability of Functions in Mathematica

3.1	Introduction	41
3.2	Limits of a Function in Mathematica	42
3.3	Continuity of a Function in Mathematic	43
3.4	Derivatives of a Function in Mathematica	43
3.5	Plotting of graphs of polynomials of degree 4 and 5	45

4 Tracing of Parametric Curves

4.1	Parametric representation of curves	55
4.2	Tracing of parametric equations	56
	4.2.1 Tracing of parametric equations in R2	56
	4.2.2 Tracing of parametric curve in Space	62
	4.2.3 Tracing of parametric surface in Space	64
4.3	Some special parametric curves	67
	4.3.1 Trochoid	67
	4.3.2 Cycloid	67
	4.3.3 Epicycloid	68
	4.3.4 Hypocycloid	69

5 Tracing of Polar Equations

5.1 Polar Equations	75
5.2 Distinguishing between Cartesian and Polar Co-ordinates	76
5.2.1 Transformation of Cartesian Co-ordinates to Polar	
Co-ordinates	77
5.2.2 Transformation of Polar Coordinates to Cartesian	
Co-ordinates	78
5.3 Tracing Polar Curves	79

Contents .

xv

6 Hyperbolic Functions in Mathematica	
6.1 Introduction6.2 Curve Tracing of Hyperbolic Functions/InverseHyperbolic Functions in Mathematica	95 97
6.3 Calculus of Hyperbolic Functions	102
7 Parabola, Ellipse and Hyperbola in Mathematica	
7.1 Introduction	113
7.2 Parabola in Mathematica	114
7.3 Ellipse in Mathematica	119
7.5 Tracing of Parabola Ellinse and Hyperbola by Using	124
Manipulation or Animation Command	129
8 Revolving Surfaces in Mathematica	
8.1 Introduction	137
8.2 Revolving Surface in Mathematica	137
9 Complex Numbers in Mathematica	
9.1 Introduction	159
9.2 Basic Operations of Complex Numbers in Mathematica	160
9.3 Graphical Representation of Complex Number	162
9.4 Solving Complex Equation in Mathematica	166
10 Finite and Infinite Subsets of \mathbb{R}	
10.1 Random Numbers	177
10.2 Generating Random Numbers in Mathematica	177
10.3 Tracing of Random Real Numbers in Mathematica	181
11 Matrices	
11.1 Introduction	189

•••	Contents
••	Contentis

xvi 😳

11.2 Operations on Matrices 11.3 Cayley Hamilton Theorem and its Verification	190
in Mathematica	195
11.4 System of Linear Equations	196
11.4.1 Homogeneous System of Linear Equations	197
11.4.2 Nonhomogeneous system of linear equations	201
12 Sequence and its Convergence in Mathematica	
12 Sequence and its Convergence in Mathematica 12.1 Sequences	219
12 Sequence and its Convergence in Mathematica 12.1 Sequences 12.2 Bounded Sequence	219 221
12 Sequence and its Convergence in Mathematica 12.1 Sequences 12.2 Bounded Sequence 12.3 Subsequence	219 221 221
 12 Sequence and its Convergence in Mathematica 12.1 Sequences 12.2 Bounded Sequence 12.3 Subsequence 12.4 Convergent Sequence 	219 221 221 221 221
12 Sequence and its Convergence in Mathematica 12.1 Sequences 12.2 Bounded Sequence 12.3 Subsequence 12.4 Convergent Sequence 12.5 Divergent Sequence	219 221 221 221 221 221

12.8 Convergence of Sequence by Plotting Its Terms

12.9 Study the Convergence/Divergence of Sequences by

222

222

224

227 231

235

13 Series in Mathematica

12.11 Cauchy Sequence

12.7 Monotonic Sequence

Finding Their Limit

12.10 Bolzano Weierstrass Theorem (BWT)

12.12 Plotting of Recursive Sequences

13.1 Introduction	247
13.2 Sequence of Partial Sums	248
13.3 Convergence/Divergence of Infinite Series by Plotting	
Sequences of Partial Sums	249
13.4 Cauchy's Root Test	252
13.5 D'Alembert's Ratio Test	257
13.6 Convergence of Series	260
Model Question Paper I	267
Model Question Paper II	269
Bibliography	271

About the Book

Calculus using Mathematica is intended for Undergraduate students as a first introduction to the Mathematica Software. The authors have drawn on their extensive experience of teaching Mathematica at the undergraduate level to create an accessible language that incorporates even the tiniest of details. This has been done to make the learning process more accessible, making this book suitable for both novices and those who want to brush up on their Mathematica abilities. The book covers many subjects, from basic arithmetic to 3D surfaces. The book's clear exposition is enhanced with both solved and unresolved examples. The authors have employed straightforward and conversational language to make the text more engaging, in addition to the numerous graphs and command explanations. The book will serve as an asset for students of all Central and other Universities who intend to use Mathematica to solve problems at any level.

Salient Features

- O Concise and easy to understand presentation of concepts.
- Written with a view to present a qualitative understanding of the subject.
- Step by Step solution to various problem have been provided to enhance perception of Mathematica.
- O Numerous graphs have been plotted for better Visual effects.

About the Authors

Dr. Pragati Gautam is an Associate Professor at Department of Mathematics, Kamala Nehru College, University of Delhi. She has completed her Doctorate from University of Delhi. Her area of research includes Fixed point theory and Topological spaces. She has published more than 30 research papers in highly reputed International journals. She is the reviewer of several research journals. Dr. Gautam has delivered several invited talks and lectures at various conferences/ workshops/AFS schools. She is presently author of books Calculus, Numerical Analysis, Analytic Geometry and Applied Algebra, Practical



Mathematica especially meant for Undergraduate students of various Universities in India and abroad.

Ms. Swapnil Verma, an alumna of IIT Delhi, is an Assistant Professor in Department of Mathematics, Kamala Nehru College, University of Delhi. She is an active researcher of faculty of Mathematical Sciences, University of Delhi. She has co-authored books *Practical Mathematica, Analytic Geometry and Applied Algebra* for undergraduates. She has published more than 13 research papers in reputed National and International Journals in the field of fixed-point theory. Her core research area is Interpolative Fixed point theory in metric spaces. She has also published International Book chapter with Nova Science Publishers, New York, U.S.A. and Taylor & Francis Group.





Sultan Chand & Sons Publishers of Standard Educational Textbooks

PUDIISNEYS OF Standard Educational 23 Daryaganj, New Delhi-110002 Phones : 011-23281876, 23277843, 23266105 Email : sultanchand74@yahoo.com info@sultanchandandsons.com Website : sultanchandandsons.com



